



Guideline on Quantifying the Value of Structural Health Information

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Structure

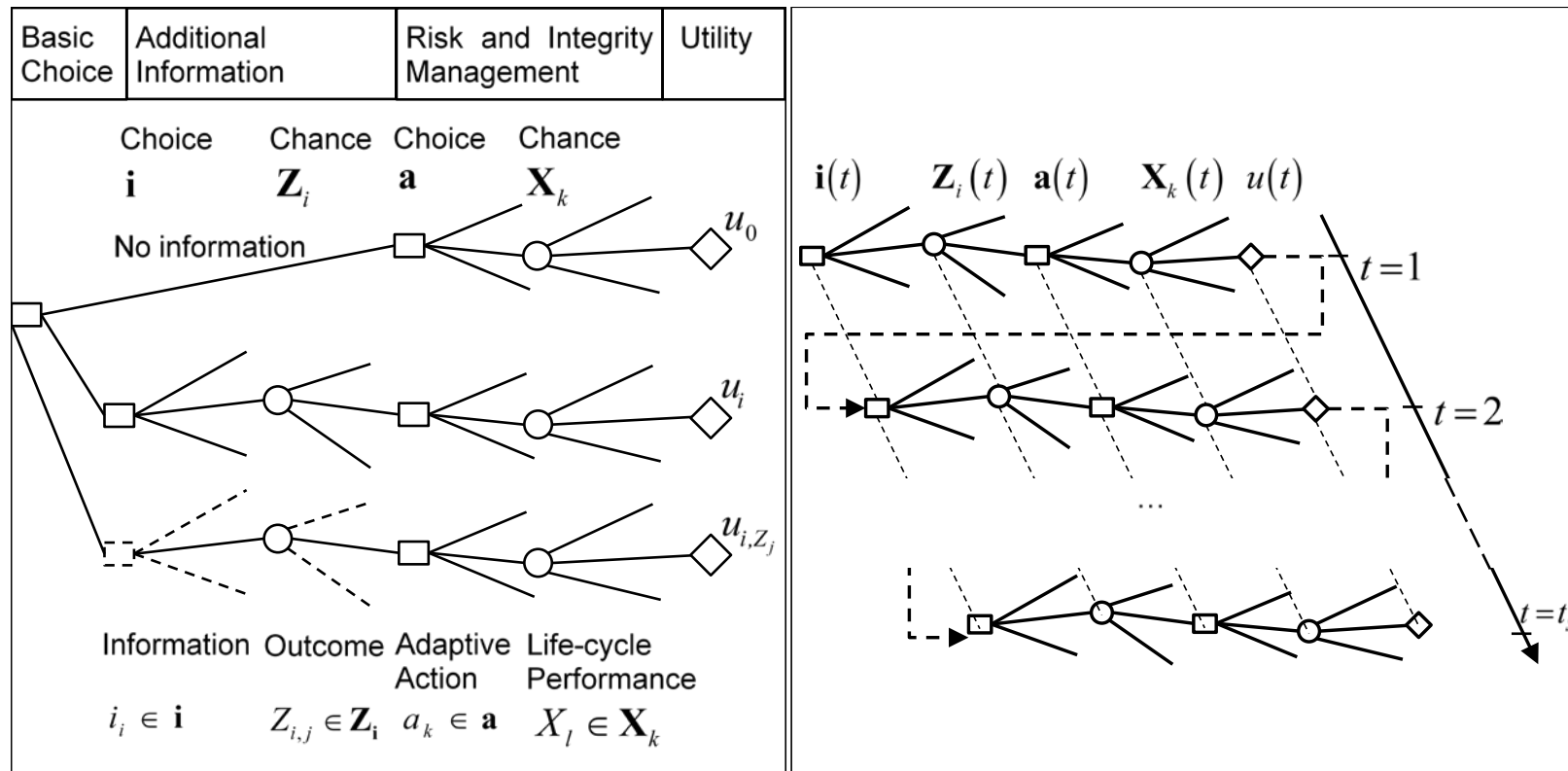
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Status

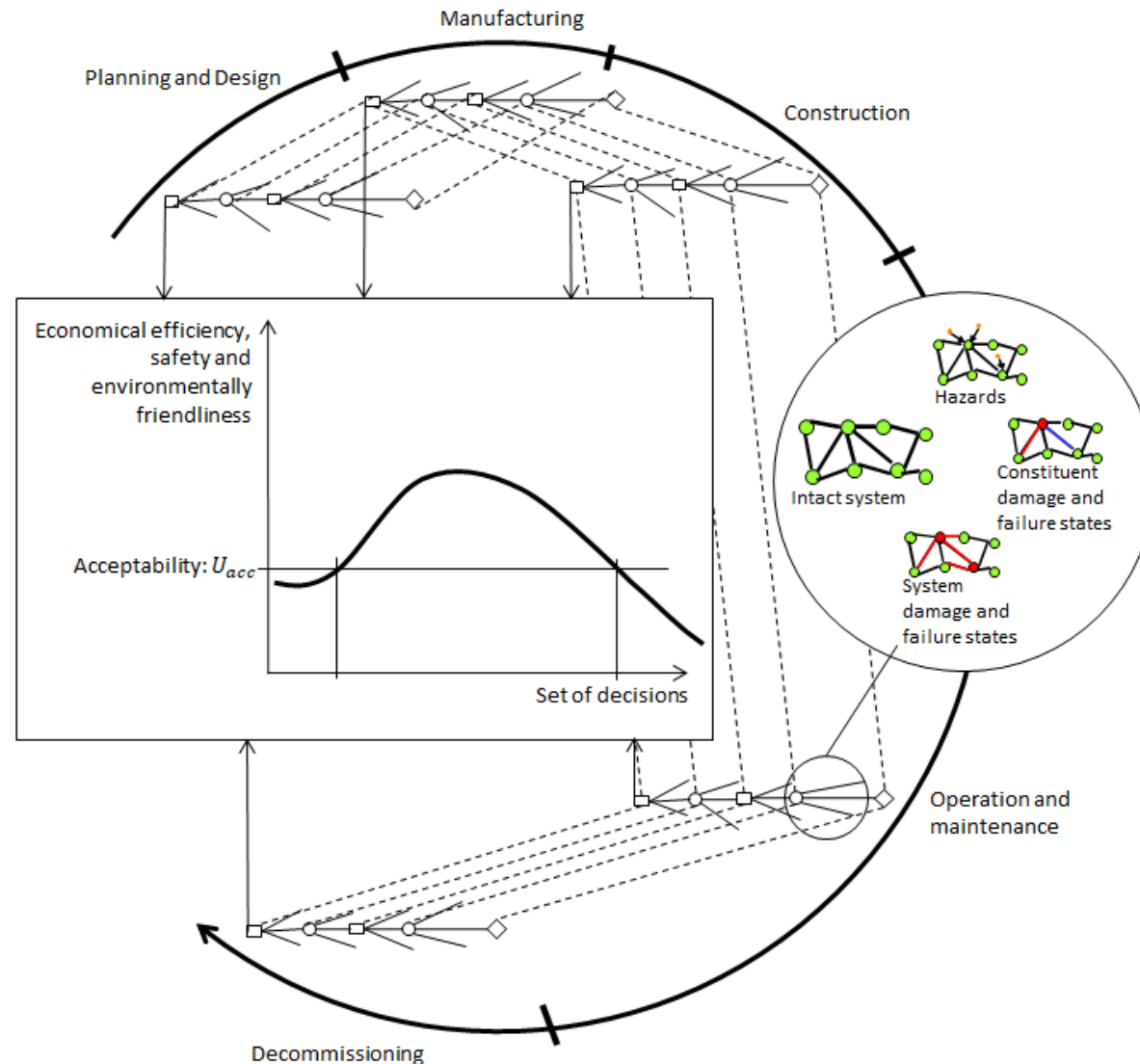


Version	Date	Pages	Changes
Built1	August 2018	9	
Draft	07.01.2019	1	Revision according to TU1402 Final Workshop, 18. and 19.10.2018, Limassol
Draft 1	18.01.2019	18	Revision according to TU1402 WG5 Workshop, 10.01.2019, Amsterdam ▪ Probabilistic characteristics of SHI extended with detection theoretical basis ▪ Minor notation alignments ▪ Information conditions added ▪ Proof loading modelling further described ▪ Detailed comments by Miroslav Sykora and Jochen Köhler and Michael Faber

Value of structural health information: decision scenario modelling and analysis



Value of structural health information: decision scenario modelling and analysis



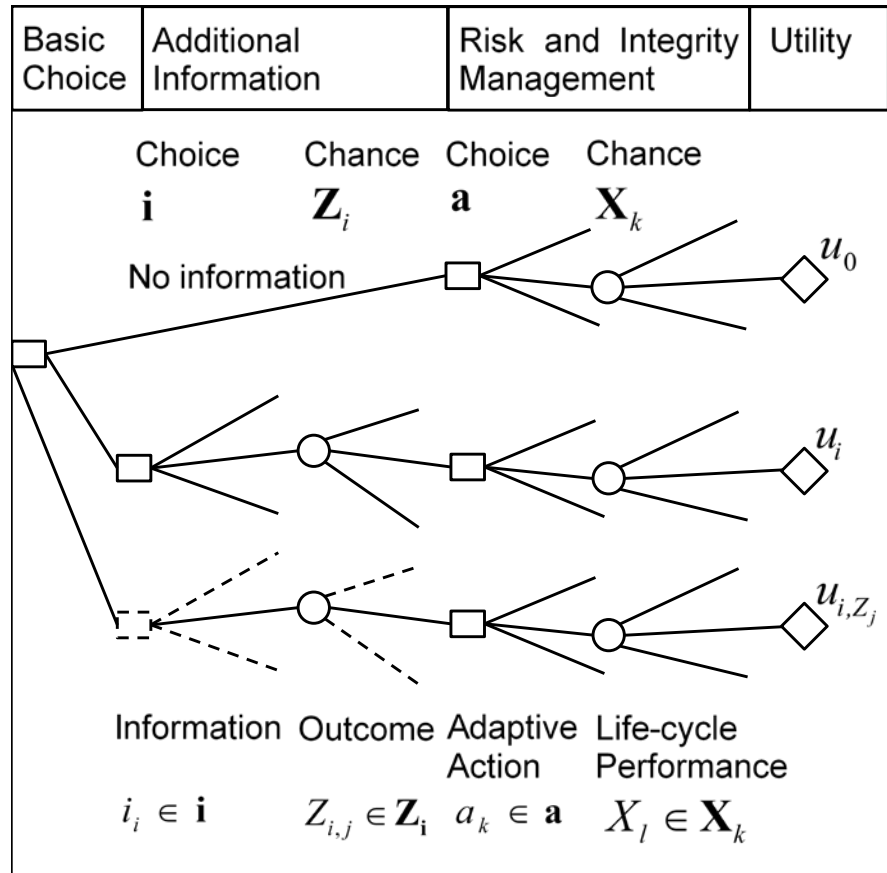


Value of structural health information: examples for typical decision scenarios

1. Utility, functionality, integrity and risk management in the operation phase of an infrastructure system
 - Integrity management planning
 - Service life extension
 - Utilisation modification
 - Functionality enhancement
 - Damage progression monitoring
 - Early damage warning
2. Code and standard calibration - decision support in the design phase
3. Structure prototype development and design by testing – design phase
4. SHI system development



Value of structural health information: objective functions



Value of SHI

$$V(i_i^*, \mathbf{a}^{*,i}, a_k^{*,0}) = U_1(i_i^*, \mathbf{a}^{*,i}) - U_0(a_k^{*,0})$$

$$\{i_i^*, \mathbf{a}^{*,i}, a_k^{*,0}\} = \arg \max_{i_i \in \mathbf{i}, \mathbf{a}^i \in \mathbf{a}} U_1(i_i, \mathbf{a}^i) - \arg \max_{a_k \in \mathbf{a}} U_0(a_k)$$

$$\text{s.t. } U_{acc} \leq U_1(i_i^*, \mathbf{a}^{*,i}) \text{ and}$$

Posterior value of SHI

$$V | Z_j = U_2(a_k^{*,i}) - U_0(a_k^{*,0})$$

$$a_k^{*,i} = \arg \max_{a_k^i \in \mathbf{a}} U_2(a_k^i) - \arg \max_{a_k^0 \in \mathbf{a}} U_0(a_k^{*,0})$$

$$\text{s.t. } U_{acc} \leq U_2(a_k^{*,i}) \quad \text{and} \quad U_{acc} \leq U_0(a_k^{*,0})$$



Structural health information

Decisions contributing to an efficient management of infrastructures rely on information conditions. According to Nielsen, Glavind et al. (2018), it can be distinguished between:

- 1) The information is relevant and precise.
- 2) The information is relevant but imprecise.
- 3) The information is irrelevant.
- 4) The information is relevant but incorrect.
- 5) The flow of information is disrupted or delayed.

From this context, it follows that structural health information is information with relevance for the decisions influencing the infrastructure performance and utility. SHI are thus required to be relevant by the temporal period and for the structural functionality, damage and failure events, their consequences and the actions. A SHI model encompasses the information type and content, the probabilistic properties and the costs and consequences. The boundaries of the probabilistic SHI models should be well documented to account for information conditions (above).



Structural health information: types

The SHI type is classified in the context of the structural system performance models. The temporal (discrete, continuous) and spatial (constituent, system) boundaries and the direct and the indirect measurements of structural performance parameters determine the SHI type.

SHI type	Temporal characteristic	Spatial characteristic	Direct / indirect
Damage detection with a distributed measurement systems and the analysis of static and dynamic behaviour	Continuous or periods of measurements	System or subsystem level	Indirect (indication of damage)
Inspection	Discrete	Constituent	Indirect (indication of damage)
Load testing	Discrete	Constituent, subsystem or system level	Indirect (indication of survival or damage or failure)
Monitoring	Continuous or periods of measurements	Constituent	Direct
Non-destructive testing	Discrete	Constituent	Direct



Structural health information: probabilistic characteristics

Probabilistic characteristics originate from:

(1) Measurement process

- (a) caused by the conversion of e.g. electrical or optical signals to structural properties and the process inherent uncertainties relating to the sensor precision, conversion and amplification unit.

(2) SHI installation and operation and

- (a) the sensor and SHI system installation
- (b) the operational conditions, which are not covered by data normalisation
- (c) human errors.

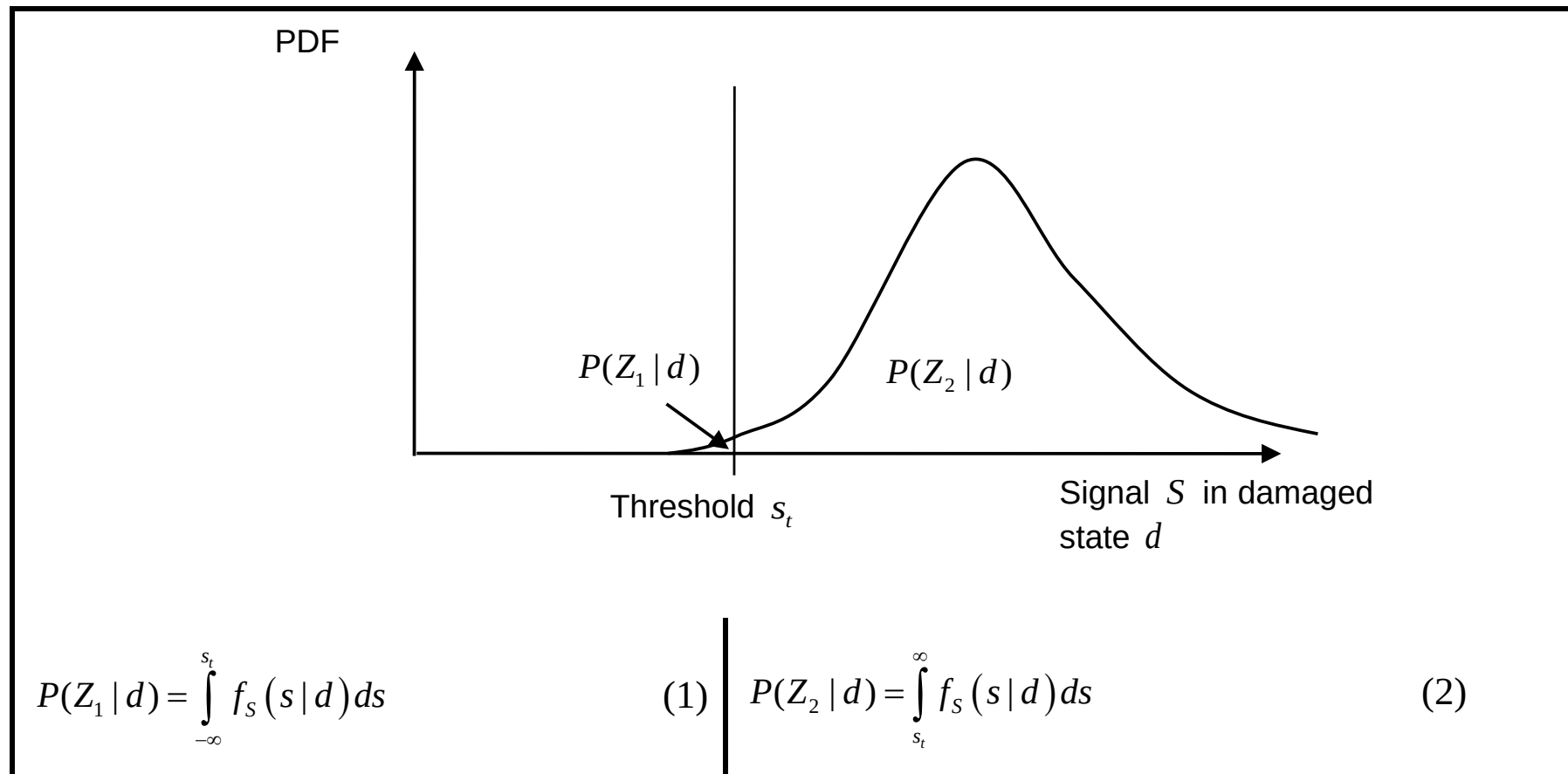
(3) SHI data analysis

- (a) the statistical uncertainties due to a limited amount of data and limited measurement periods in relation to the temporal boundaries of the decision scenario,
- (b) the limited precision of data analysis and data normalisation algorithms
- (c) human errors.
- (d) For consecutive or multiple SHI, the dependency characteristics must be explicitly modelled.



Structural health information: probabilistic characteristics

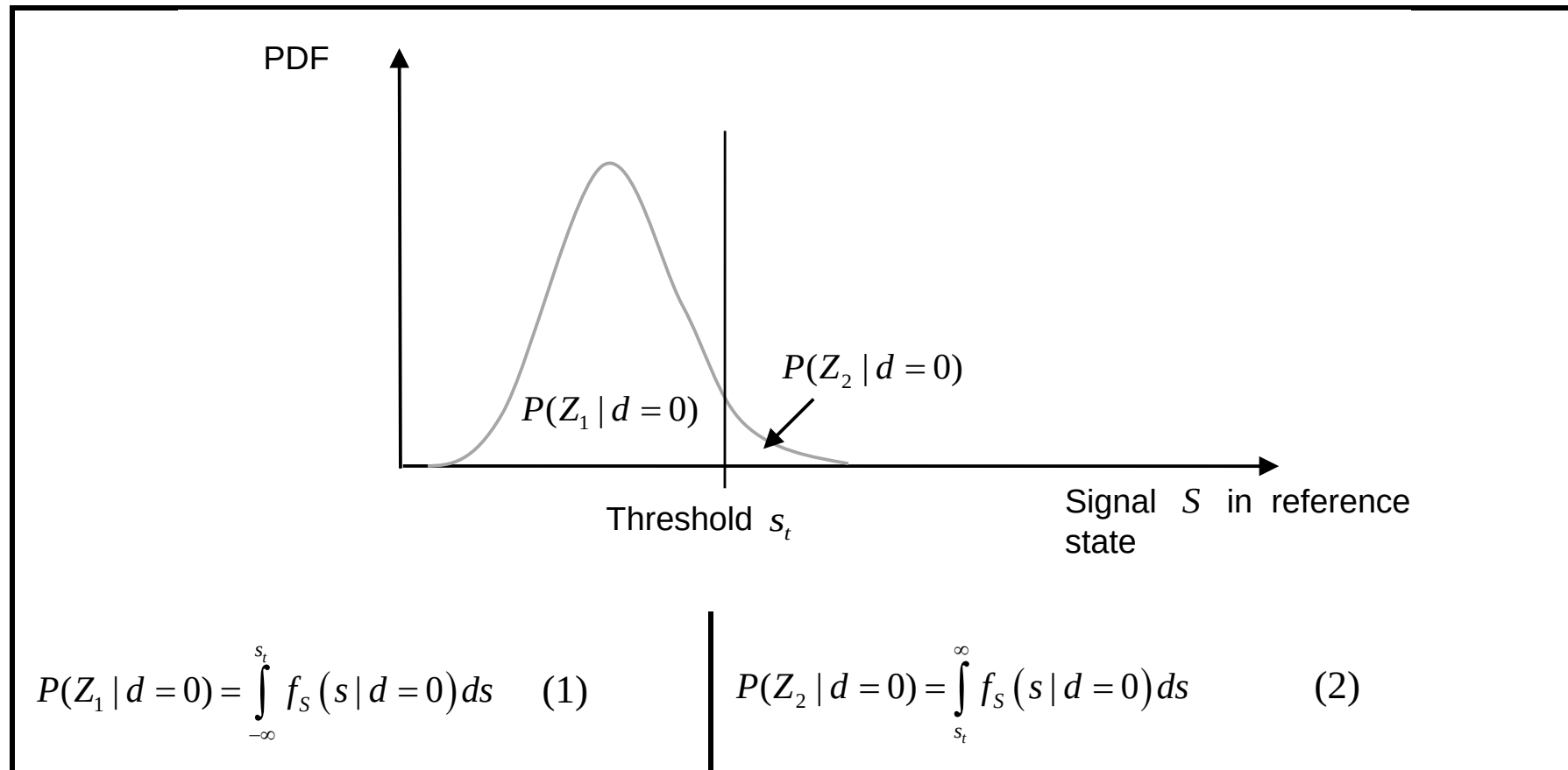
Signal processing: detection theory





Structural health information: probabilistic characteristics

Signal processing: detection theory





Structural health information: probabilistic characteristics

Signal processing: detection theory

$$P(Z_1(d=0, \mathbf{d})) = 1 - P(Z_2(d=0, \mathbf{d}))$$

No detection and detection of constituent damage states

$$P(Z_{1,S}(\mathbf{d}_{S,0}, \mathbf{d}_S)) = 1 - P(Z_2(\mathbf{d}_{S,0}, \mathbf{d}_S))$$

No detection and detection of system damage states

$$P(Z_{2,S} | \mathbf{d}_{S,0}, \mathbf{d}_S) = \int_{s_t}^{-\infty} f_{n_c, S}(s | \mathbf{d}_{S,0}, \mathbf{d}_S) ds$$

Marginal detection and no detection probabilities with n_C variate signal distributions

$$P(Z_{1,S} | \mathbf{d}_{S,0}, \mathbf{d}_S) = \int_{-\infty}^{s_t} f_{n_c, S}(s | \mathbf{d}_{S,0}, \mathbf{d}_S) ds$$



Structural performance adaptation and structural health information modelling: Damage detection

$$P(X_{1,c} | \mathbf{Z})P(\mathbf{Z}) \\ = P(\mathbf{Z} | X_{1,c})P(X_{1,c}) = P(\mathbf{Z} \cap X_{1,c})$$

Pre-posterior probability of the constituent state for a value of SHI analysis

$$P(X_{1,c}(t) | Z_{j_1}(t_{i,1}) \dots Z_{j_n}(t_{i,n})) \\ = \frac{P(X_{1,c}(t) \cap Z_{j_1}(t_{i,1}) \cap \dots \cap Z_{j_n}(t_{i,n}))}{P(Z_{j_1}(t_{i,1}) \cap \dots \cap Z_{j_n}(t_{i,n}))}$$

Posterior probability of the constituent state for a posterior value of SHI analysis with multiple indications over time

$$X_{1,c} : g_{X_{1,c}}(\mathbf{X}_{X_{1,c}}) = M_{R_c} R_c(D_c) - M_{S_c} S_c \leq 0$$

$$Z_j : g_{Z_j}(\mathbf{X}_{Z_j}) = P(Z_j(D_c)) - U \leq 0$$

Limit state functions for constituents

- Resistance R_c , damage D_c and corresponding model uncertainties M_{R_c} and M_{S_c} , constituent damage size dependent probability of indication curve $I_j(D_c)$, uniformly distributed random variable U



Damage detection and load testing

$$X_{1,S} : \left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ M_{R_{i_c,j_c}} R_{i_c,j_c} (D_{i_c,j_c}) - M_{S_{i_c,j_c}} S_{i_c,j_c} \leq 0 \right\} \right)$$

Limit state functions for systems with a system constituents damage size dependent probability of indication curve $I_S(D_c)$

$$Z_{1,S} : g_{Z_{1,S}}(\mathbf{X}_{Z_{1,S}}) = P(Z_{1,S}(\mathbf{D})) - U \leq 0$$

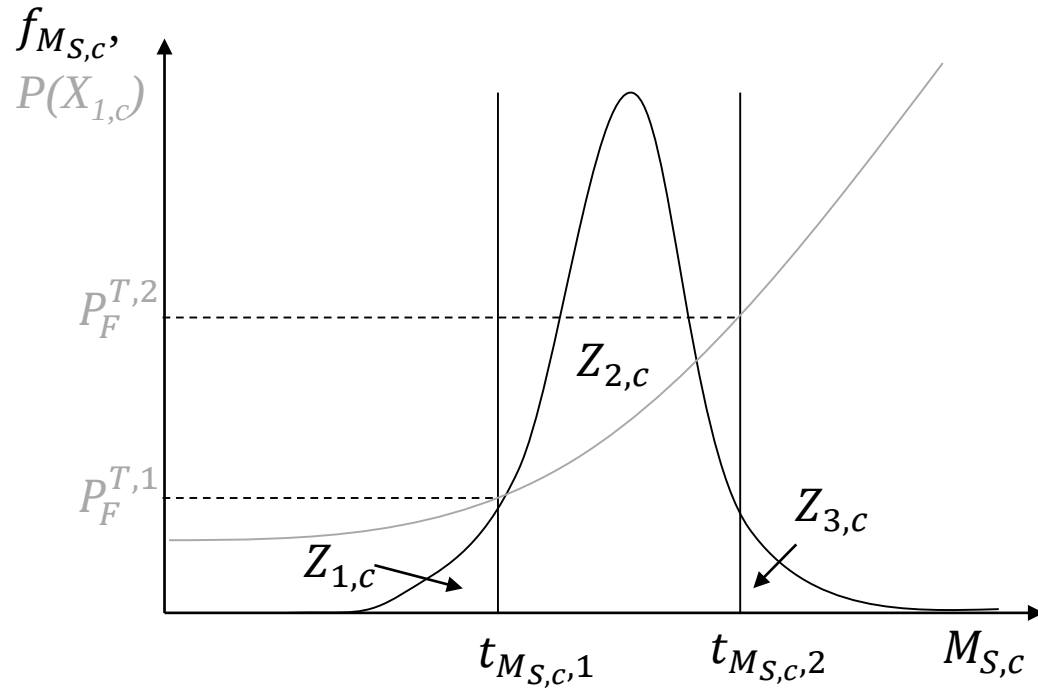
$$Z_{1,c} : g_{Z_{1,c}}(\mathbf{X}_{Z_{1,c}}) = M_{R_c} R_c(D_c) - M_{S_{c,T}} S_{c,T} > 0$$

Limit state functions for constituent and system load testing ($S_{C,T}$ and $S_{ic,jc,T}$)

$$Z_{1,S} : \left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ M_{R_{i_c,j_c}} R_{i_c,j_c} (D_{i_c,j_c}) - M_{S_{i_c,j_c,T}} S_{i_c,j_c,T} \leq 0 \right\} \right)$$



Monitoring information modelling



Monitoring information (temporal continuous, spatial discrete) may be modelled by exploiting the model uncertainty determination.

- Thresholds $t_{M_{S,c},1}$ and $t_{M_{S,c},2}$ can be derived with target probabilities $P_F^{T,1}$ and $P_F^{T,2}$
- Probabilities of indication are derived with threshold truncated distributions.

$$Z_{1,c} : P(Z_{1,c}) = \int_{-\infty}^{t_{M_{S,c},1}} f_{M_{S,c}}(m_{S,c}) dm_{S,c}$$

$$Z_{2,c} : P(Z_{2,c}) = \int_{t_{M_{S,c},1}}^{t_{M_{S,c},2}} f_{M_{S,c}}(m_{S,c}) dm_{S,c}$$

$$Z_{3,c} : P(Z_{3,c}) = \int_{t_{M_{S,c},2}}^{\infty} f_{M_{S,c}}(m_{S,c}) dm_{S,c}$$



Monitoring information modelling

$$g_{X_{1,c}, Z_{2,c}}(\mathbf{X}_{X_{1,c}}, Z_{2,c}) \\ = M_{R_c} R_c(D_c) - M_{S,c} \left[t_{M_{S,c},1}; t_{M_{S,c},2} \right] M_U S_c \leq 0$$

$$g_{X_{1,c}, Z_{2,c}}(\mathbf{X}_{X_{1,c}}, Z_{2,c}) \\ = M_{R_c} R_c(D_c) - m_{S,c} M_U S_c \leq 0$$

Pre-posterior probability of the constituent state for a value of SHI analysis

- Truncated distribution of $M_{S,c}$ and monitoring uncertainty M_U

Posterior probability of the constituent state for a posterior value of SHI analysis

- Realisation $m_{S,c}$ and monitoring uncertainty M_U



Equality information modelling

$$Z_{1,S} : g_{Z_{1,S}}(\mathbf{X}_{Z_{1,S}}, M_U) = M_U - m(\mathbf{X}_{X_{1,S}}) \leq 0$$

Equality information modelling

Limit state function representing the (equality) measurement information may be derived with the uncertain measurement M_U and the prior distribution of the measurement value $m(\mathbf{X}_{X_{1,S}})$.

Limit state functions for direct calculation of $P(X_{1,c}(t) \cap \mathbf{Z}(t_{i,1}) \cap \dots \cap \mathbf{Z}(t_{i,n}))$ may be derived.

Next steps: principal example





Thank you for your attention.