

JCSS Continuing Education & Advanced School

Structural Reliability and Probabilistic Model Code

Target Reliability II:

Design and assessment criteria for safety and cost efficiency

Jochen Köhler

Norwegian University of Science and Technology

Overview

- General performance criteria for the build environment
- Self-contained approach to assess performance
- Simplification = Generalisation
- How safe is safe enough? => A calibration problem!

The build environment

- As the main contributor to our societal development,
- And, as a major consumer of natural resources,
- Needs proper **strategies for decision support** for further development and maintenance !!
- Objective: sustainable development.

The build environment



Strategy

- Decisions are made
- It is not how we can identify the right decision, but how we identify the “best” decision
- Reasonable to assess the effect of different decision alternatives on “our” utility

Formal Decision Theory

Actions

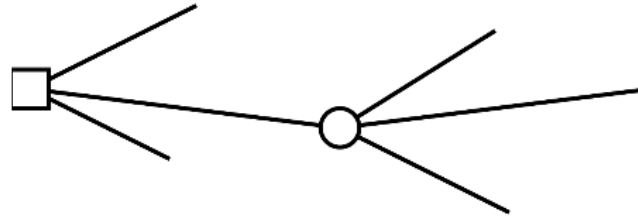
$a \in A$

State

$\theta \in \Theta$

Utility

$u(.)$



What can I know?

What should I do?

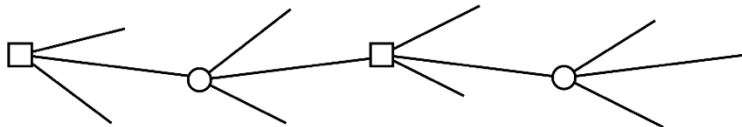
What may I hope?

- Reasonable strategy
- Challenging to apply
- Simplifications necessary

System definition

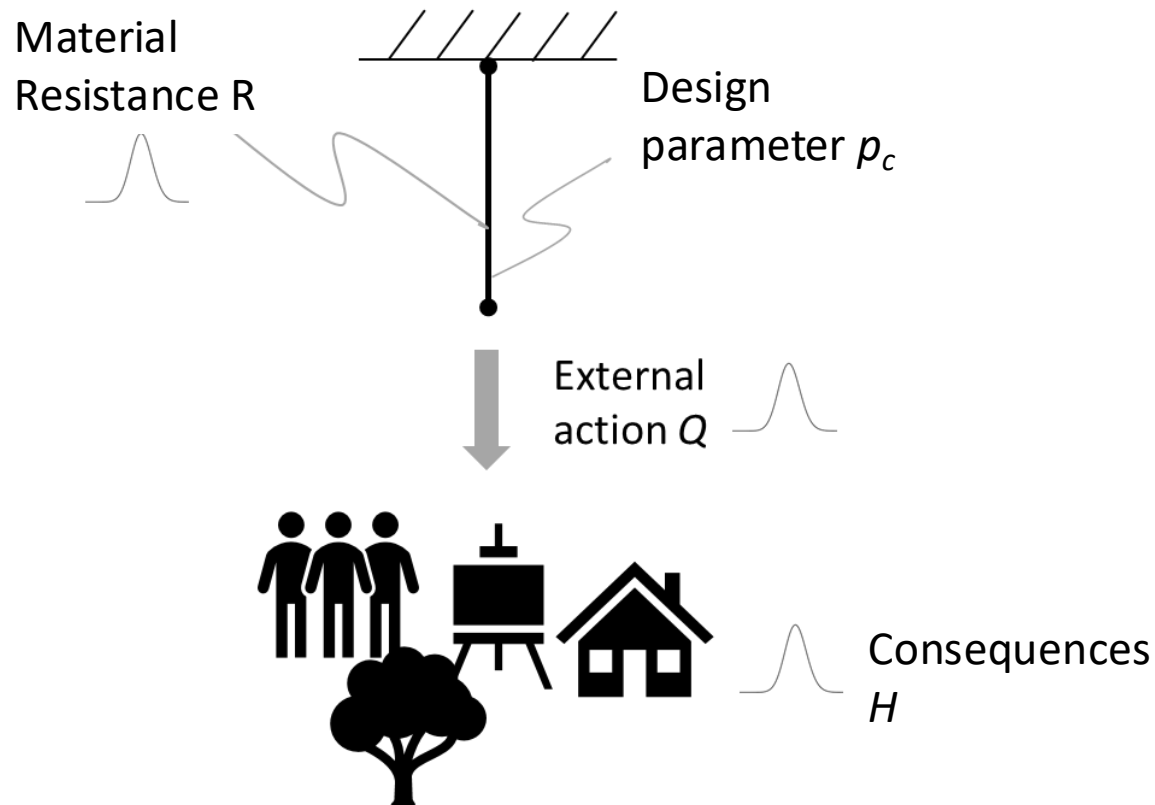
- Reduction and simplification
- Decision alternatives $\leftrightarrow$ utility

<u>Experiment</u>	<u>Sample</u>	<u>Actions</u>	<u>State</u>	<u>Utility</u>
$e \in E$	$z \in Z$	$a \in A$	$\theta \in \Theta$	$u(.)$

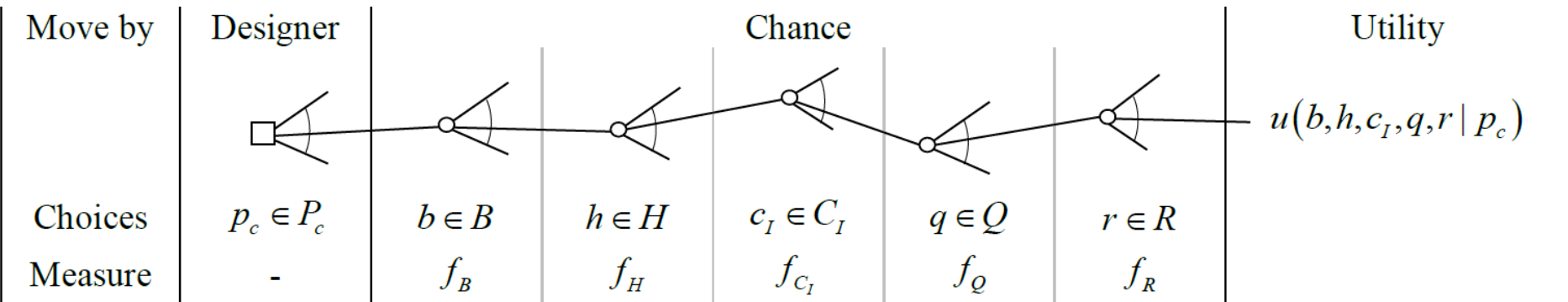


Structural design decision problem

- Objective: minimum use of resources over time



Structural design decision

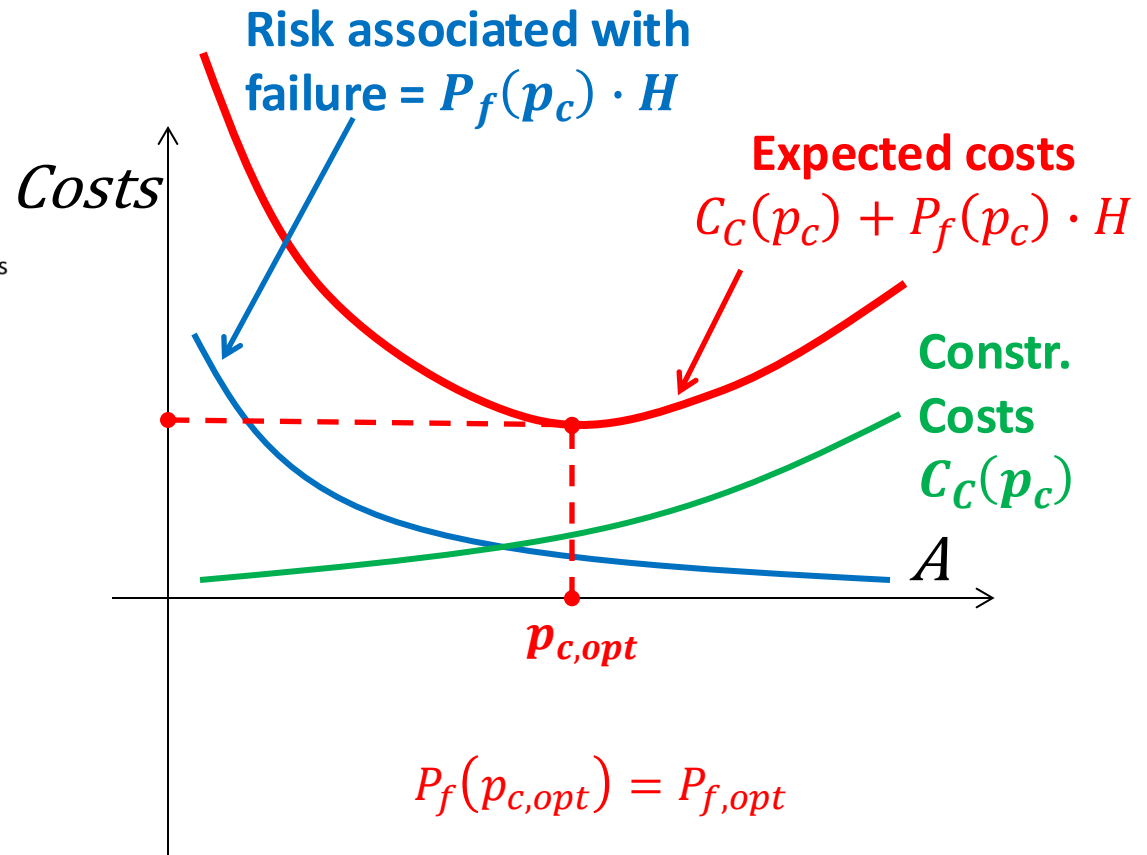
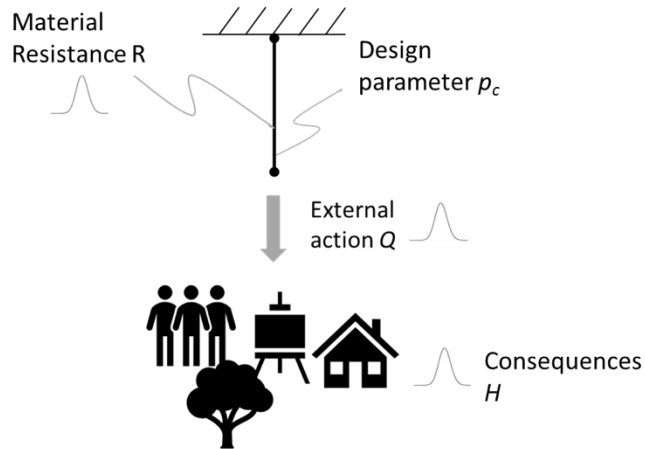


$$\mathbf{p}_{c,opt} = \underset{\mathbf{p}_c}{\operatorname{argmax}} \{E_{\Theta}[u(\Theta, \mathbf{p}_c)]\} = \underset{\mathbf{p}_c}{\operatorname{argmin}} \{E_{\Theta}[C_{tot}(\Theta, \mathbf{p}_c)]\}$$

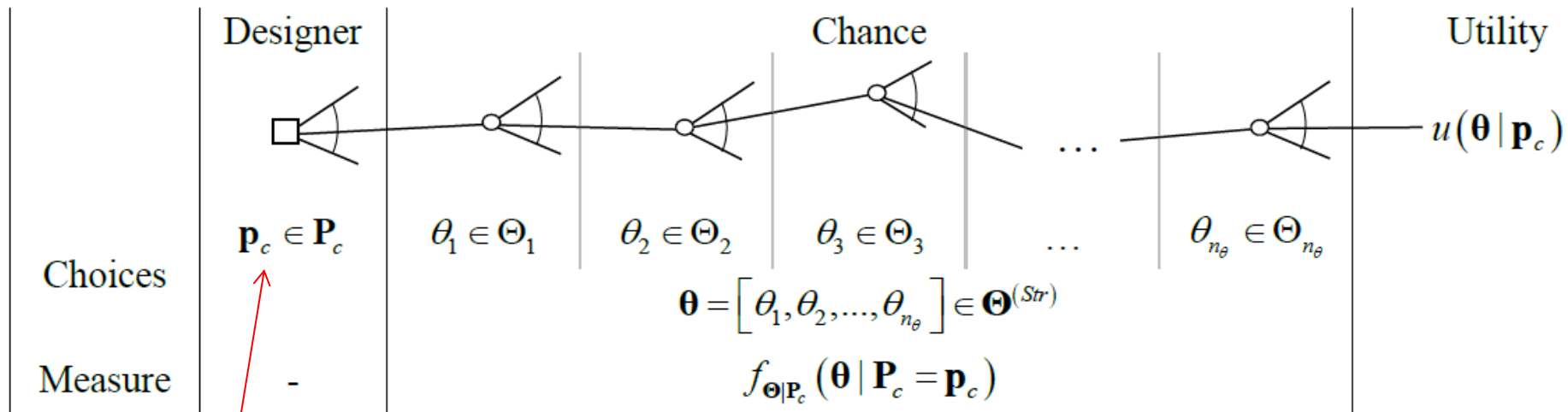
$$E_{\Theta}[C_{tot}(\Theta, \mathbf{p}_c)] = (E[C_0] + E[C_1]p_c) - E[H]P_f(\mathbf{p}_c)$$

$$P_f(\mathbf{p}_c) = \int_{p_c r < q} f_{R,Q}(r, q) dr dq$$

Risk informed decision



Generalization of the risk informed design problem



Decision variable

"State of the world" Variables
Not known with certainty by the decision maker

Level of detail
sensibly chosen!

$$\mathbf{p}_{c,opt} = \underset{\mathbf{p}_c}{\operatorname{argmax}} \{E_{\Theta}[u(\mathbf{\theta} | \mathbf{p}_c)]\}$$

Simplified design methods

Approaches:

Risk-informed

Decisions taken considering full risk
(Level 4 design)

Reliability-based

Decisions taken with reliability
requirement to fulfil (Level 3 and 2 design)

Semi-probabilistic

Safety format prescribing the design
equations and/or analysis for
assessing decisions (Level 1 design)

Simplifications:

None

Avoid explicit evaluation of
failure consequences/ safety
costs etc.

Avoid explicit evaluation of
failure consequences/ safety
costs etc. AND avoid
reliability analyses

Objective:

*Minimise use of societal resources
over time*

Target reliability index or Pf

Partial safety factors, modification
factors, load reduction factors etc.

Reliability elements in standards:

Reliability-based calibration

Risk-based calibration

Application domains

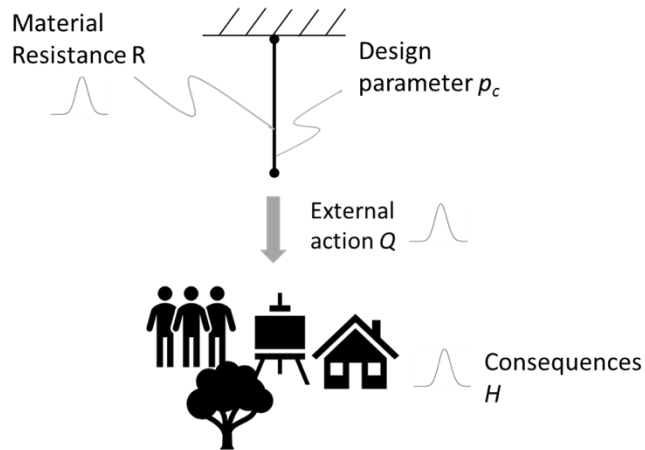
Simplified design and assessment of decision approaches [ISO 2394]

- **Level 4: Risk-informed**

- **Levels 3 (and 2): Reliability-based**



Reliability based design (Level 3 and 2)



Design:

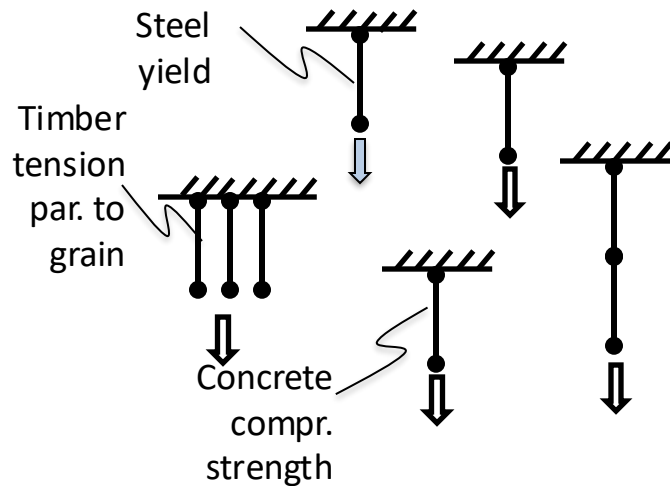
$$p_c: P_f(p_c) = P_{f,target}$$

$$\text{Level 3} \equiv \text{Level 4} \Leftrightarrow P_{f,target} \equiv P_{f,opt}$$

Code calibration, why?

- Simplification:
 1. No explicit evaluation of costs, consequences, etc

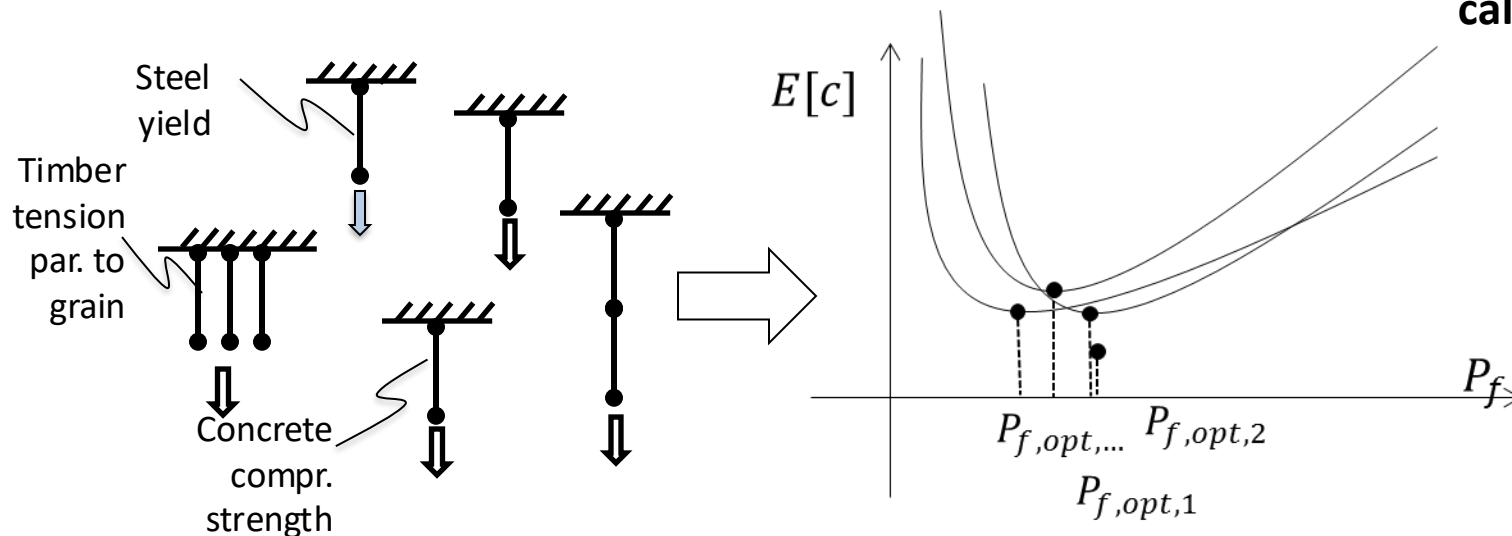
→ simplify calculations



Code calibration, why?

- Simplification:
 1. No explicit evaluation of costs, consequences, etc

→ simplify calculations

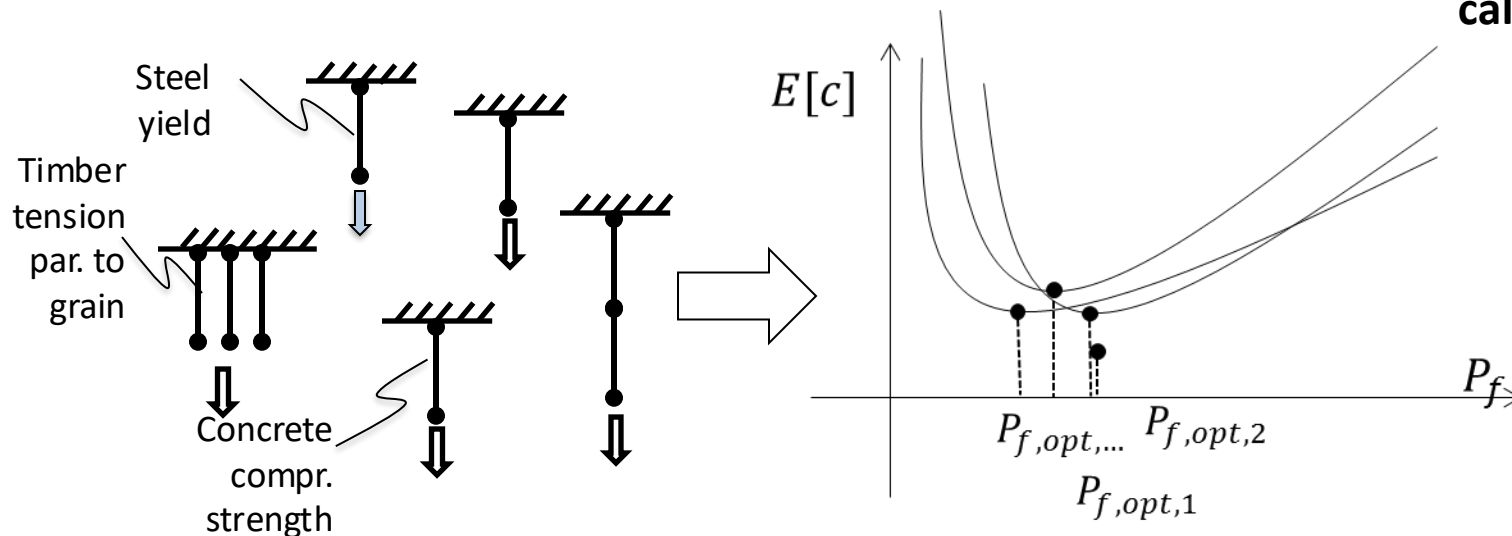


Code calibration, why?

- Simplification:

1. No explicit evaluation of costs, consequences, etc

→ simplify calculations



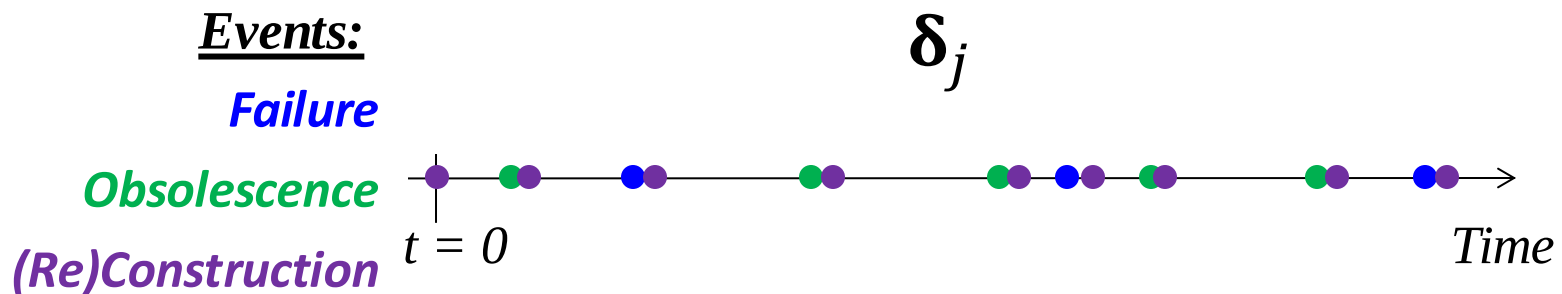
2. One $P_{f,target}$ for a class of structures

→ simplify standards and calculations

CALIBRATION: what $P_{f,target}$ is optimal for the class?

Code calibration as a decision problem under risk

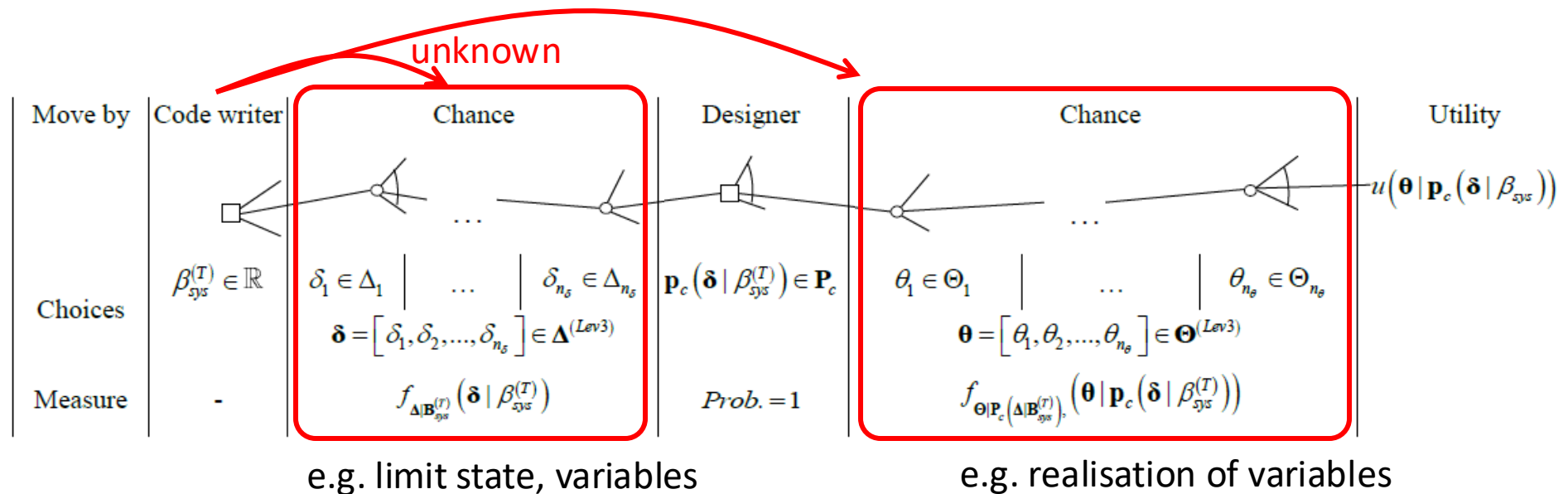
- Decision variable: β_{target} for Level 3 and 2 design
 - each structure in the class defined by δ
 - present and future structures



- Decision maker: society (codes guard the interest of society)
- Level of detail in system representation consistent with the generalisation over classes

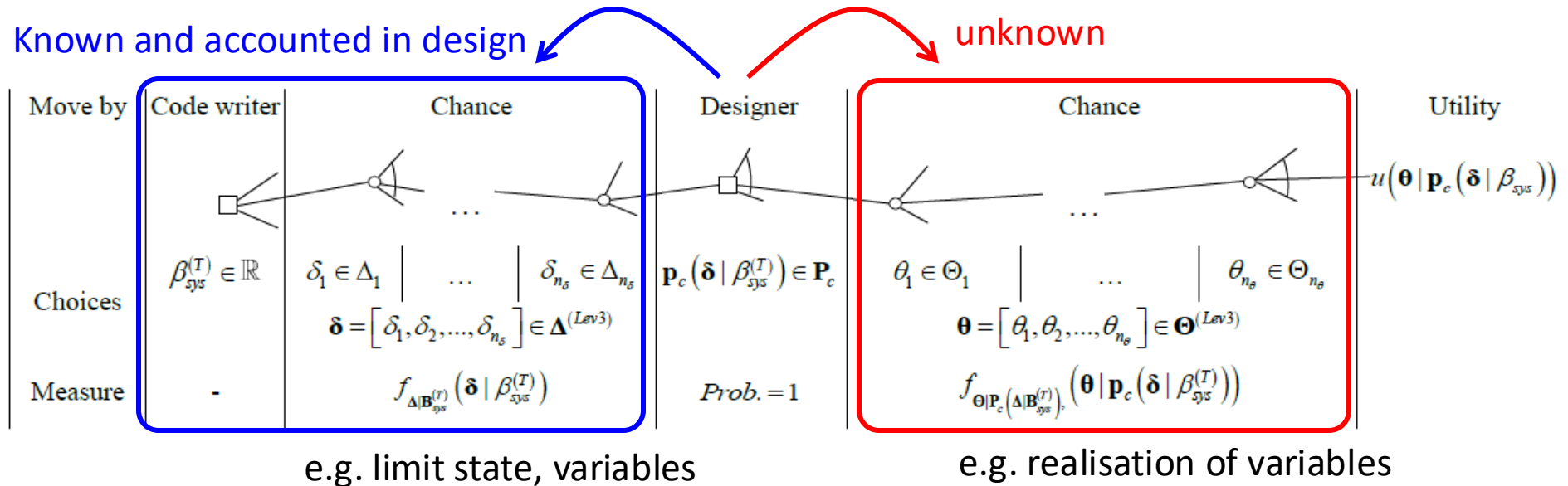
Optimisation of β_t for Level 3 codes

- Game between *Code writer* and *Chance*
 - Code writer* selects a β_t
 - Chance* chooses a possible structure to be designed $\delta \in \Delta^{(Lev3)}$
 - Designer finds dimensions $\mathbf{p}_c$ giving $\beta \equiv \beta_t$
 - Chance* chooses a state of the nature $\theta \in \Theta^{(Lev3)}$



Optimisation of $\beta_{sys,t}$ for Level 3 codes

- Game between *Code writer* and *Chance*
 - Code writer* selects a β_t
 - Chance* chooses a possible structure to be designed $\delta \in \Delta^{(Lev3)}$
 - Designer finds dimensions $\mathbf{p}_c$ giving $\beta \equiv \beta_t$
 - Chance* chooses a state of the nature $\theta \in \Theta^{(Lev3)}$



Current target reliability values in JCSS PMC and ISO 2394

- Based on monetary optimization

		Failure consequences		
		Minor	Moderate	Large
Relative cost of safety	Large	3.1 ($P_f \approx 10^{-3}$)	3.3 ($P_f \approx 5 \cdot 10^{-4}$)	3.7 ($P_f \approx 10^{-4}$)
	Normal	3.7 ($P_f \approx 10^{-4}$)	4.2 ($P_f \approx 10^{-5}$)	4.4 ($P_f \approx 5 \cdot 10^{-6}$)
	Small	4.2 ($P_f \approx 10^{-5}$)	4.4 ($P_f \approx 5 \cdot 10^{-6}$)	4.7 ($P_f \approx 10^{-6}$)

- Risk optimisation philosophy included by differentiation of consequences and cost for safety.
- Differentiation is coarse - > consistent with level of information.
- But qualification into classes is difficult.

Background Reliability Target Table

- Objective function

$$\begin{aligned} E [C_{tot} (p)] &= C_{constr} (p) + E [C_f (p)] \frac{1}{\gamma} + E [C_{obs} (p)] \frac{1}{\gamma} \\ &= [C_0 + C_I p] + [C_0 + C_I p + H] \frac{\lambda P_f^{(1a)} (p)}{\gamma} + [C_0 + C_I p + D] \frac{\omega}{\gamma} \end{aligned}$$

- Yearly probability of failure based on the simple $R - S$ problem.
- The variability of R and S chosen such that it represents the characteristics of a class of structures.

Background Reliability Target Table

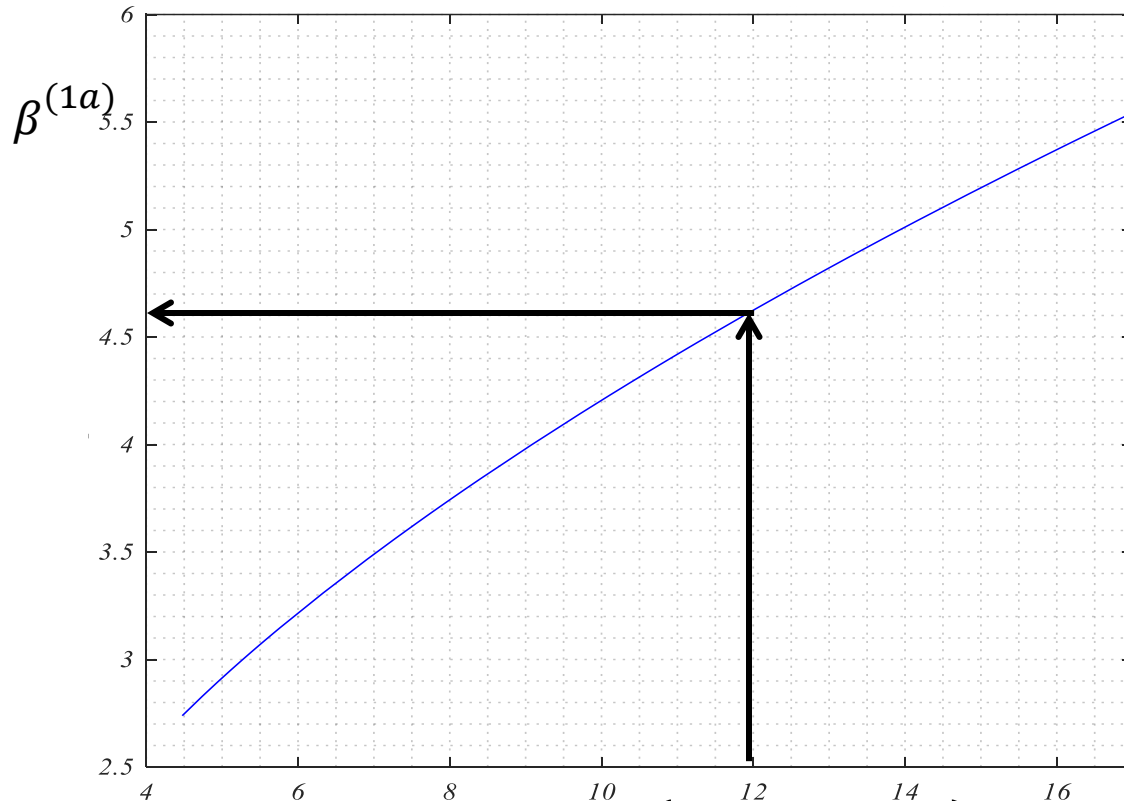
- Optimisation

$$\frac{d}{dp} \left\{ C_0 + C_I p + [C_0 + C_I p + H] \frac{\lambda P_f^{(1a)}(p)}{\gamma} + [C_0 + C_I p + D] \frac{\omega}{\gamma} \right\} \bigg|_{p=p^*} \equiv 0$$

$$\Rightarrow \frac{C_0 + C_I p^* + H}{C_I} = \frac{1 + P_f^{(1a)}(p^*) \frac{1}{\gamma} + \frac{\omega}{\gamma}}{-\frac{dP_f^{(1a)}(p)}{dp} \bigg|_{p=p^*} \frac{1}{\gamma}}$$

- Reordering and simplification:
$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + H} \approx -\frac{dP_f^{(1a)}(p^*)}{dp} \bigg|_{p=p^*}$$

Plot representing target reliabilities



Line satisfying the condition at optimum

$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + H} \approx - \frac{dP_f^{(1a)}(p^*)}{dp} \bigg|_{p=p^*}$$

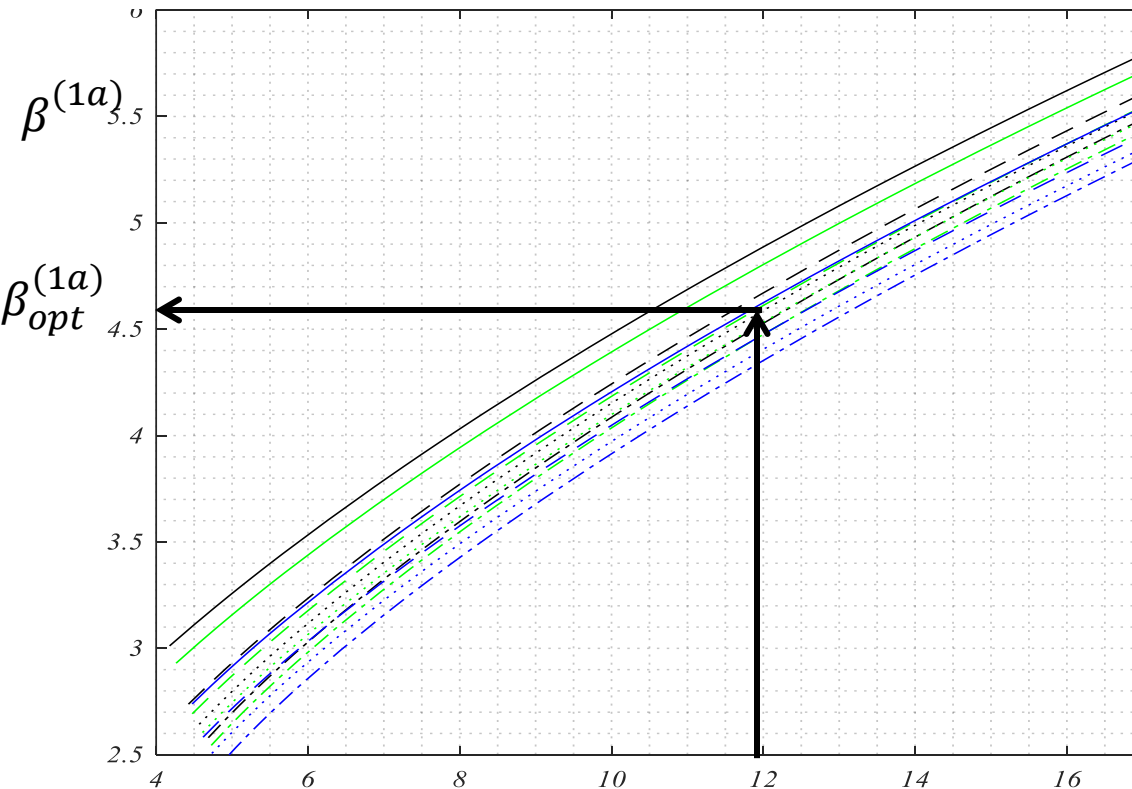
for:

$$COV_R = 0.15 \text{ and } COV_S = 0.30$$

$$\ln \left\{ \frac{C_0 + H}{C_I(\gamma^{(1a)} + \omega)} \right\}$$

Safety costs;
Failure costs;
Interest rate γ ;
Obsolescence rate ω .

Plot representing target reliabilities



Different types of uncertainties

- $V_R=0.05, V_S=0.1$
- - - $V_R=0.05, V_S=0.3$
- $V_R=0.05, V_S=0.45$
- - - - $V_R=0.05, V_S=0.6$
- $V_R=0.15, V_S=0.1$
- - - $V_R=0.15, V_S=0.3$
- $V_R=0.15, V_S=0.45$
- - - - $V_R=0.15, V_S=0.6$
- $V_R=0.3, V_S=0.1$
- - - $V_R=0.3, V_S=0.3$
- $V_R=0.3, V_S=0.45$
- - - - $V_R=0.3, V_S=0.6$

$$\ln \left\{ \frac{C_0 + H}{C_I \gamma^{(1a)} + \omega} \right\}$$

Safety costs;
Failure costs;
Interest rate γ ;
Obsolescence rate ω .

Life Safety

- The reliability requirement, so far, was based on optimisation.
- Our societal preferences for life safety can not be related to potential benefit of a economic endeavour!
- On the other hand, additional reliability is obtained by investing more monetary means.
- Societal willingness to pay (SWTP): How much **can** a society invest to reduce the fatality rate in structures?

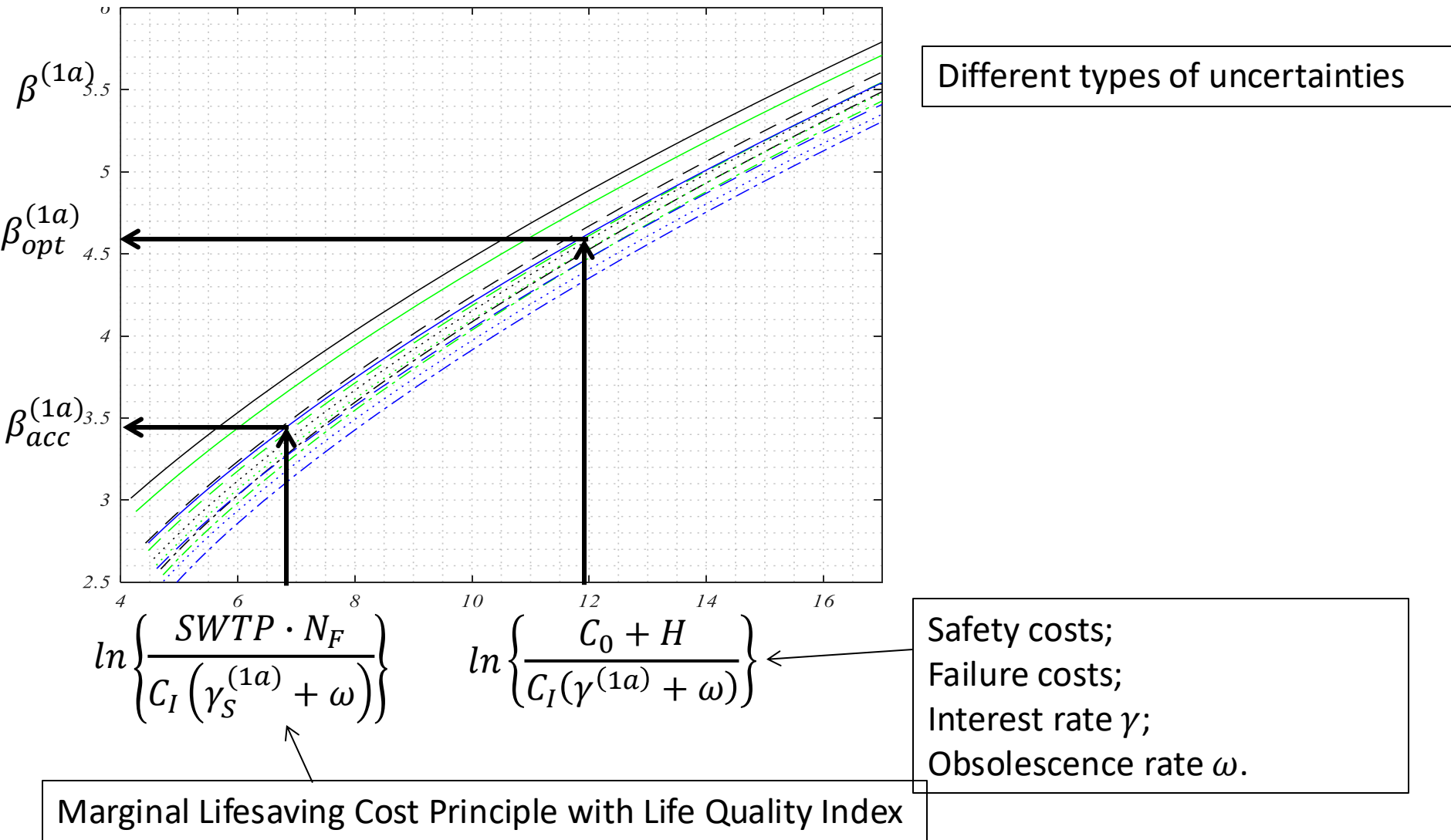
Life Safety – modified objective

$$\frac{d}{dp} \left\{ C_0 + C_I p + \textcolor{red}{N_F SWTP} \frac{\lambda P_f^{(1a)}(p)}{\gamma} + [C_0 + C_I p + D] \frac{\omega}{\gamma} \right\} \bigg|_{p=p^*} \equiv 0$$

- Correspondingly it has to be invested at least:

$$-\frac{dP_f^{(1a)}(p)}{dp} \leq \frac{C_I (\gamma_S + \omega)}{SWTP \cdot N_F} = K_1$$

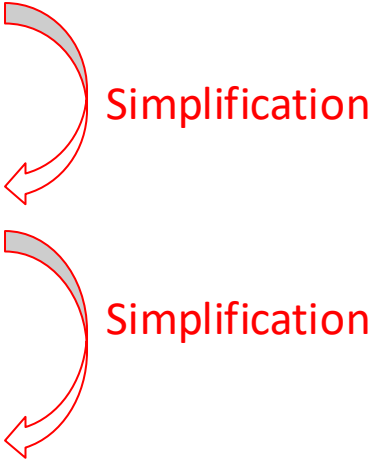
Plot representing target reliabilities



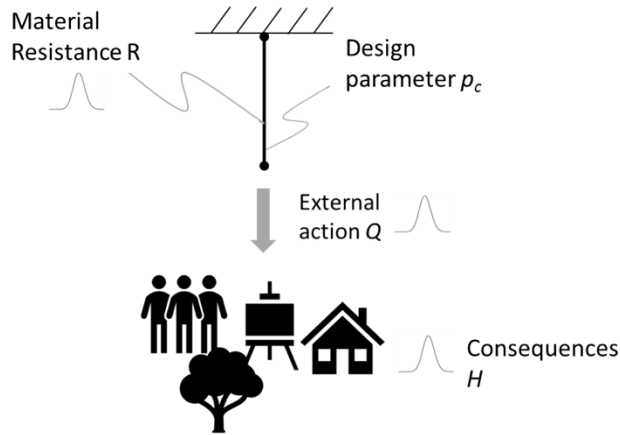
Summary

- Determination of target reliabilities for reliability based design is a calibration problem
- Generalisation and classification requires “low” level of detail of system representation
- Risk criteria can be in-cooperated
- Risk based design is open to any/(the appropriate) level of detail.

Simplified design and assessment of decision approaches [ISO 2394]

- **Level 4: Risk-informed**
 - **Levels 3 (and 2): Reliability-based**
 - **Level 1: Semi-probabilistic**
- 
- The diagram illustrates a process of simplification from Level 4 to Level 1. It features two red curved arrows pointing downwards. The top arrow starts at the level of 'Level 4: Risk-informed' and points to 'Levels 3 (and 2): Reliability-based', with the word 'Simplification' written in red to its right. The bottom arrow starts at the level of 'Levels 3 (and 2): Reliability-based' and points to 'Level 1: Semi-probabilistic', also with the word 'Simplification' written in red to its right.

Semi-probabilistic approach (Level 1)



Partial Safety Factors (reliability elements)

Design:

$$p_c: p_c \geq \frac{\gamma_M}{f_k} \cdot \gamma_Q \cdot q_k$$

$$\text{Level 1} \equiv \text{Level 4} \Leftrightarrow \gamma_M, \gamma_Q: P_f \left(p_c = \frac{\gamma_M}{f_k} \cdot \gamma_Q \cdot q_k \right) \equiv P_{f,opt}$$

Code calibration, why?

- Simplification:

1. No explicit evaluation costs, consequences, etc.
2. No reliability analyses

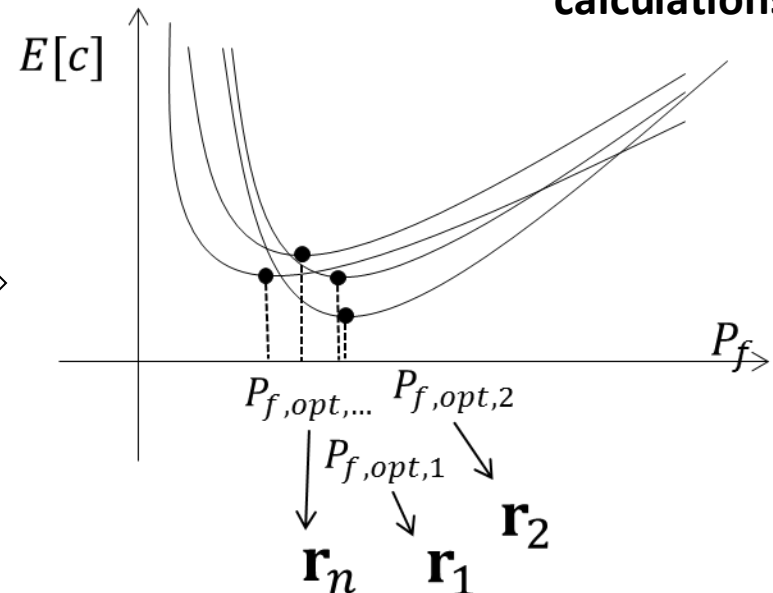
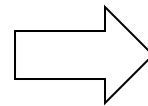
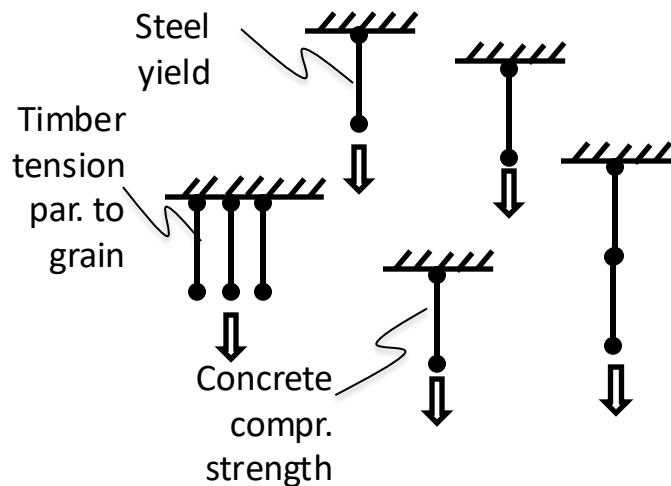
→ simplify
calculations

Code calibration, why?

- Simplification:

1. No explicit evaluation costs, consequences, etc.
2. No reliability analyses

→ simplify calculations

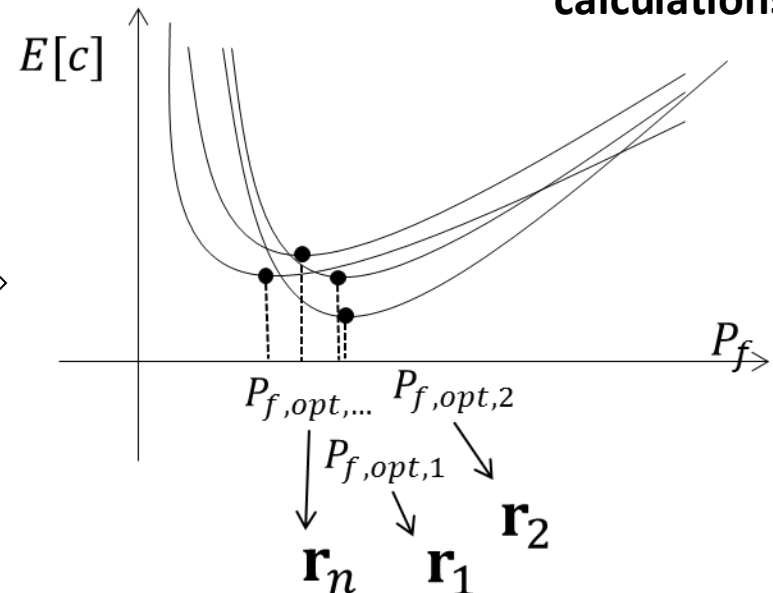
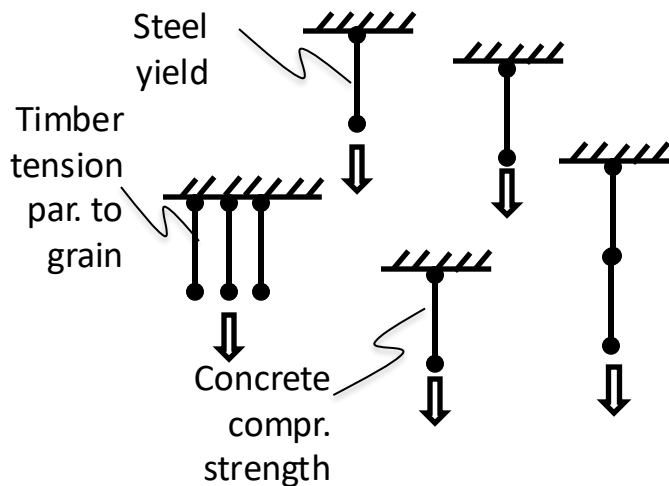


Code calibration, why?

- Simplification:

1. No explicit evaluation costs, consequences, etc.
2. No reliability analyses

→ simplify calculations



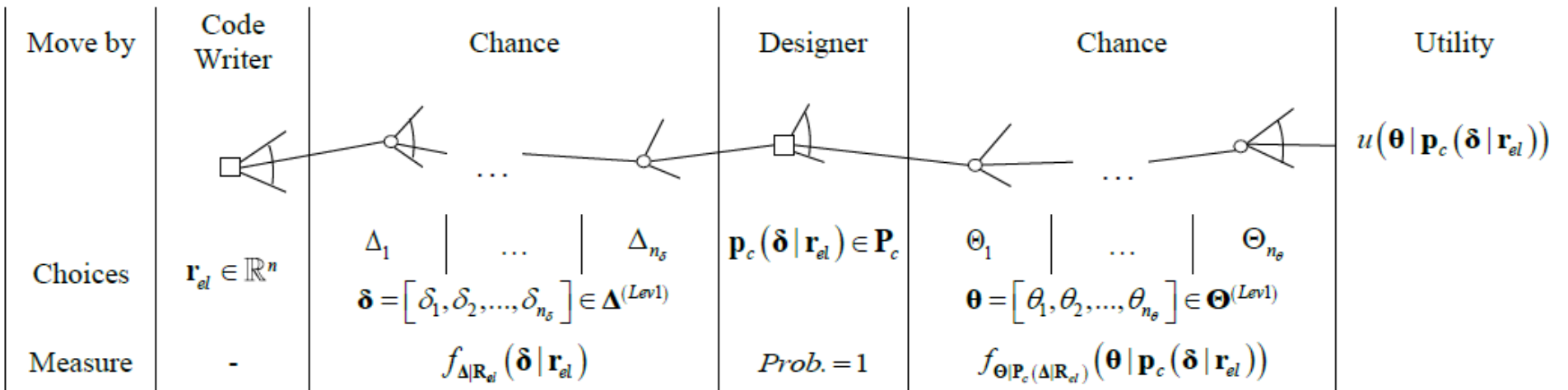
3. One $\mathbf{r} = [\gamma, \psi_0, k_{mod}]$ for a class of structures

→ simplify standards and calculations

CALIBRATION: what $\mathbf{r}$ is optimal for the class?

Decision problem

- Decision variable: $\mathbf{r}_{el}$ for Level 1 design for a class of structures
 - Partial safety factors
 - Modification factors
 - Load combination factors

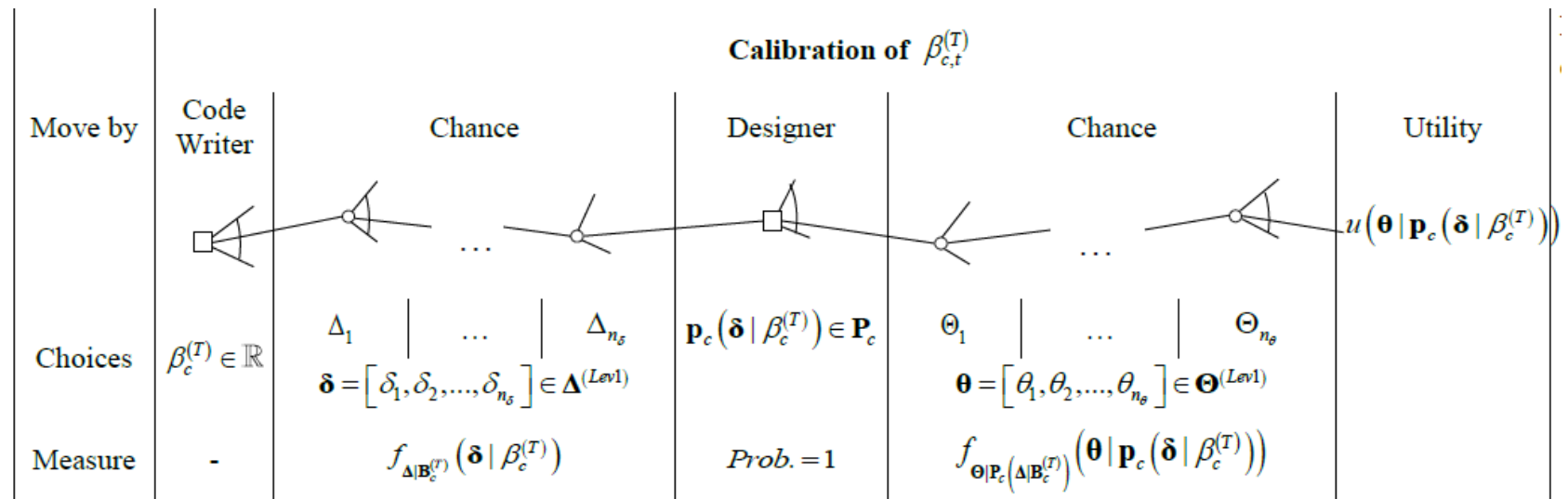


e.g. type of structure

Simplified decision problem

1. Optimise $\beta_{c,target}$

- Decision variables: $\beta_{c,target}$ for Level 1 design for a class of structures



2. Reliability-based calibration

- $\mathbf{r}_{el,opt}: \beta_c(\mathbf{r}_{el})$ as close as possible to $\beta_{c,target} = \beta_{c,opt}$

Code Calibration Overview

