JCSS Meeting-WP1-Ongoing works for concrete working group

Concrete Properties in JCSS Code and Possible Improvements

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December 8, 2021

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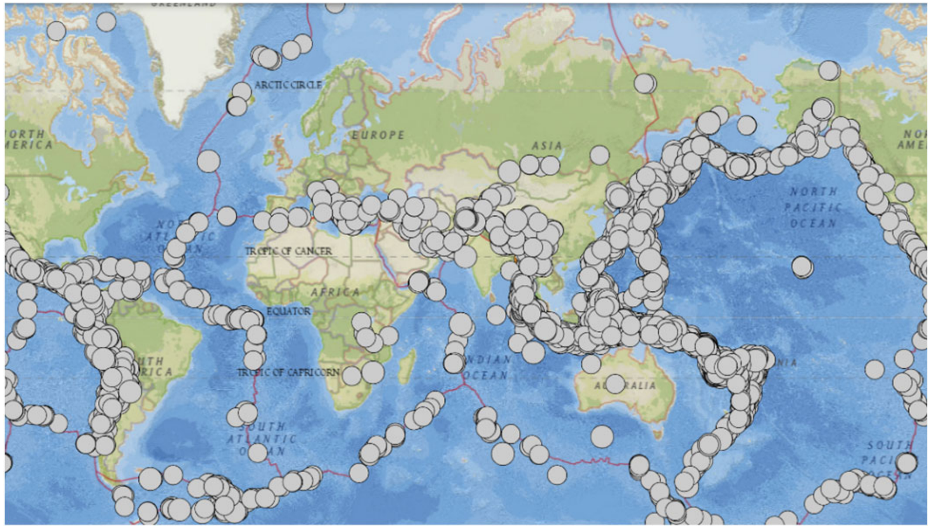
Contents

1 Background

- The damage, destruction and collapse of concrete structures under major disasters are the key causes of casualties and property losses.



Since 2000, 3343 earthquakes with magnitude greater than 6

1971 San Fernando Earthquake

2008 Wenchuan earthquake

2010 Yushu earthquake

2011 earthquake off the Pacific coast of Tōhoku

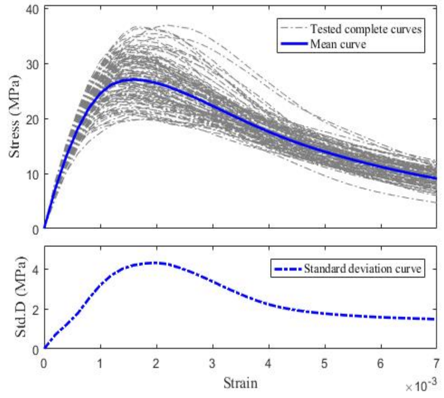
https://earthquake.usgs.gov/

1 Background

- **Stochastic damage constitutive model**

$$\sigma = (\mathbb{I} - \mathbb{D}):\bar{\sigma}$$
$$= (\mathbb{I} - \mathbb{D}):\mathbb{E}_0:(\varepsilon - \varepsilon^p)$$

$$\mathbb{D} = d^+\mathbb{P}^+ + d^-\mathbb{P}^-$$

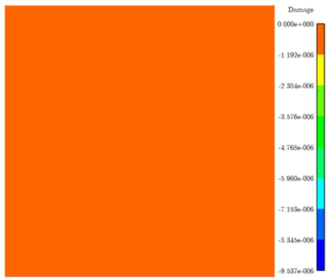


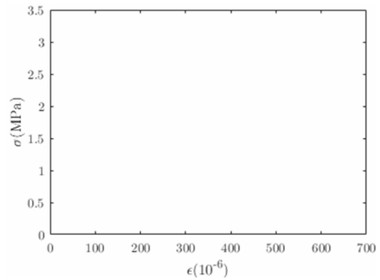
Complete uniaxial compressive stress-strain curves

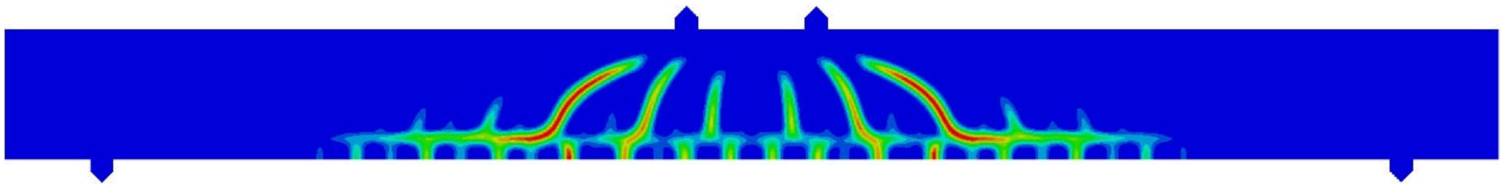
Nonlinearity and Randomness

- **Nonlocal Macro-Meso-Scale Consistent Damage Model**

- **Unified phase-field theory**



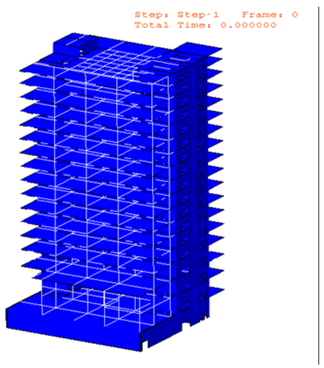




Li J Ren XD. International Journal of Solids and Structures, 2009, 46(11-12): 2407-2419.
Lu GD, Chen JB. Computer Methods in Applied Mechanics and Engineering, 2020, 362: 112802.
Wu JY. Journal of the Mechanics and Physics of Solids, 2017, 103: 72-99.

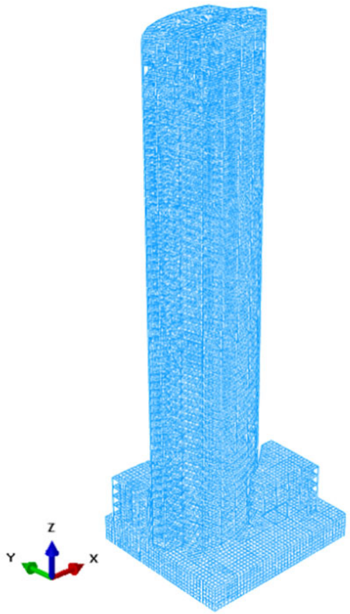
1 Background

- All kinds of complex high-rise building structures and their models



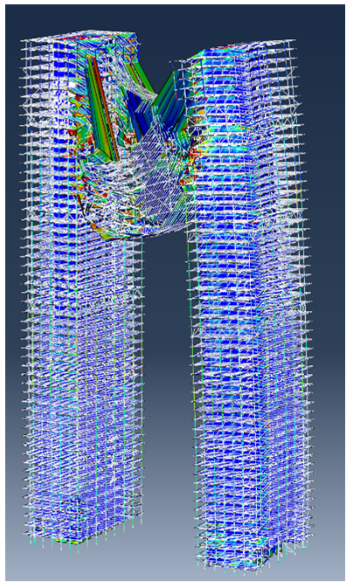
Height: 86.4 m

Elements: 53k



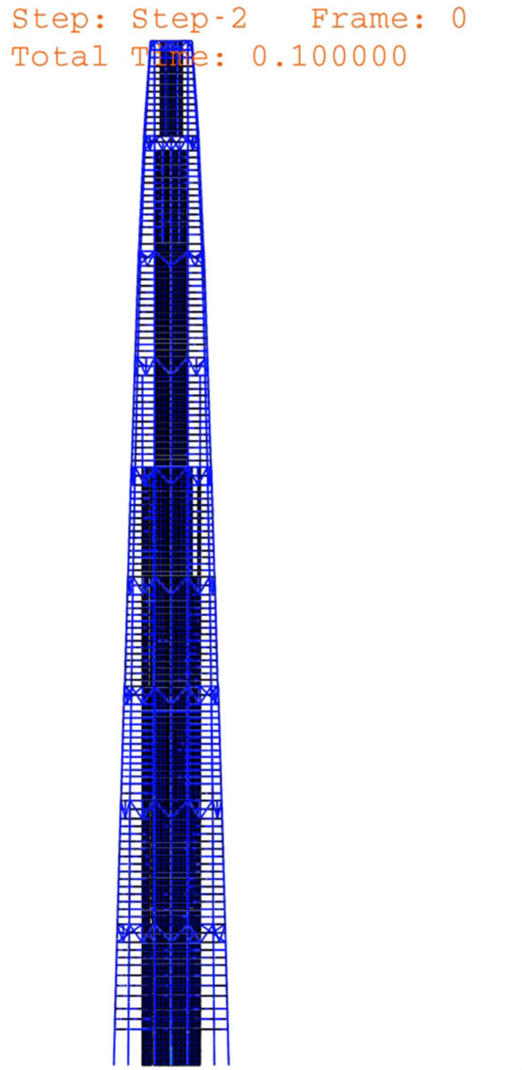
Height: 269.2 m

Elements: 85k



Height: 238.8 m

Elements: 160k



Height: 600 m

Elements: 500k

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Contents

2.1 JCSS Code

Concrete properties

In situ compressive strength: $f_c = \alpha(t,\tau) f_{c0}^{\lambda}$

Tensile strength: $f_{ct} = 0.3 f_c^{2/3}$

Modulus of elasticity: $E_c = 10.5 f_c^{1/3} \left[1 + \beta_d \varphi(t,\tau)\right]^{-1}$

Ultimate compression strain: $\varepsilon_u = 6 \times 10^{-3} f_c^{-1/6} \left[1 + \beta_d \varphi(t,\tau)\right]$

Where:

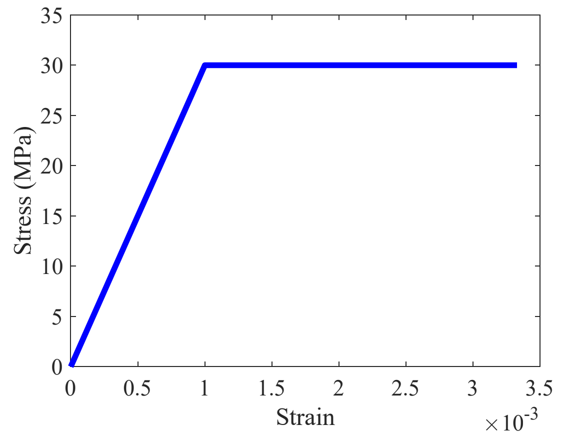
f_{c0} the compressive strength of standard test specimens;
λ the systematic variation of in situ compressive strength;
$\alpha(t, \tau)$ the concrete age at the loading time and the duration of loading;
$\varphi(t, \tau)$ the creep coefficient;
β_d the ratio of the permanent load to the total load.

2.1 JCSS Code

➢ Concrete properties

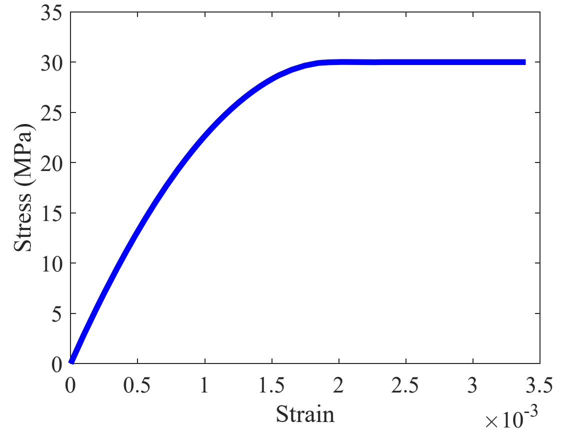
☐ Simplified stress-strain relationship:

$$\sigma=\begin{cases} E_c\varepsilon \text{ for } \varepsilon<\varepsilon_e \\ f_c \text{ for } \varepsilon_e\leqslant\varepsilon<\varepsilon_u \end{cases} \quad \text{with } \varepsilon_e = f_c / E_c$$



☐ The stress-strain relationships for the design of cross-sections:

$$\sigma=\begin{cases} f_c\left[1-\left(1-\dfrac{\varepsilon}{\varepsilon_s}\right)^k\right], \text{ for } 0<\varepsilon<\varepsilon_s \\ f_c \text{ , for } \varepsilon\geqslant\varepsilon_s \end{cases}$$



where $\varepsilon_s=0.0011\times f_c^{1/6}$, $k=E_c\varepsilon_s/f_c$.

2.1 JCSS Code

The probabilistic model

The strength of concrete at a **particular point *i*** in a given **structure *j***:

$$f_{c,ij} = \alpha(t,\tau) f_{c0,ij}^{\lambda} Y_{1,j}$$

where $Y_{1,j}$ is a log-normal variable representing **additional variations**,

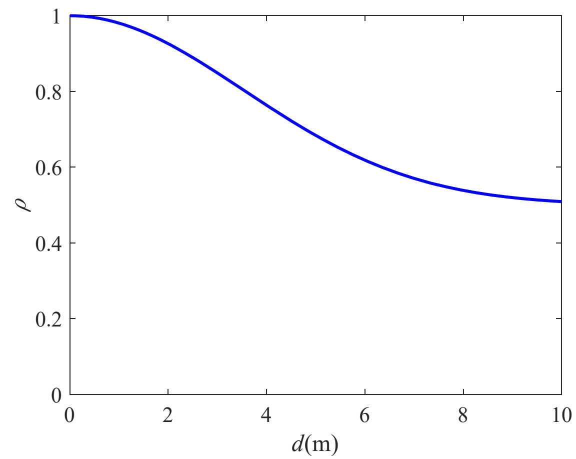
$f_{c0,ij}$ is log-normal variable:

$$f_{c0,ij} = \exp\left(U_{ij}\Sigma_j + M_j\right) \qquad f_{c0,ij} = \exp\left[m'' + t_{v''}s''\left(1+\frac{1}{n''}\right)^{0.5}\right]$$

where U_{ij} is a standard normal variable representing **the variability within one structure.**

The variables U_{ij} and U_{kj} **within one member**:

$$\rho\left(U_{ij},U_{kj}\right) = \rho + (1-\rho)\exp\left[-\frac{\left(r_{ij}-r_{kj}\right)^2}{d_c^2}\right]$$



2.1 JCSS Code

➢ The probabilistic model

The other three basic properties:

$$f_{\mathrm{ct},ij} = 0.3 f_{\mathrm{c},ij}^{2/3} Y_{2,j}$$

$$E_{\mathrm{c},ij} = 10.5 f_{\mathrm{c},ij}^{1/3} \left[1 + \beta_{\mathrm{d}} \varphi(t,\tau)\right]^{-1} Y_{3,j}$$

$$\varepsilon_{\mathrm{u},ij} = 6.10^{-3} f_{\mathrm{c},ij}^{-1/6} \left[1 + \beta_{\mathrm{d}} \varphi(t,\tau)\right] Y_{4,j}$$

where $Y_{2,j}$ to $Y_{4,j}$ reflect variations due to factors **not well accounted for by concrete compressive strength**.

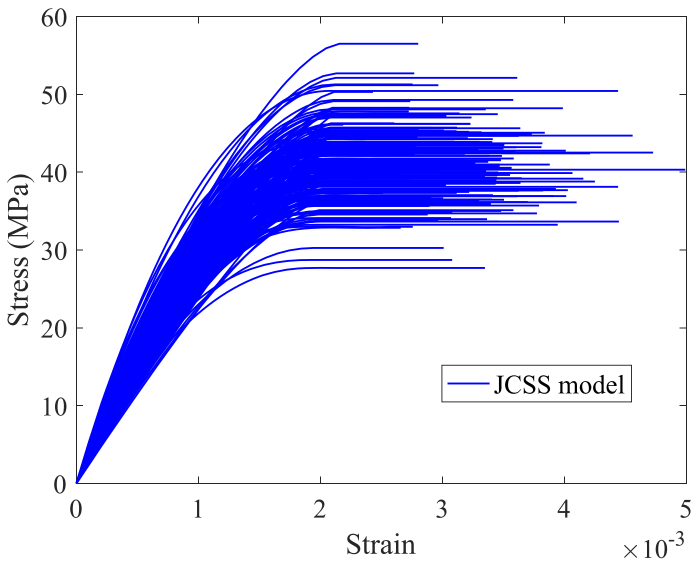
Data for parameters $Y_{k,j}$

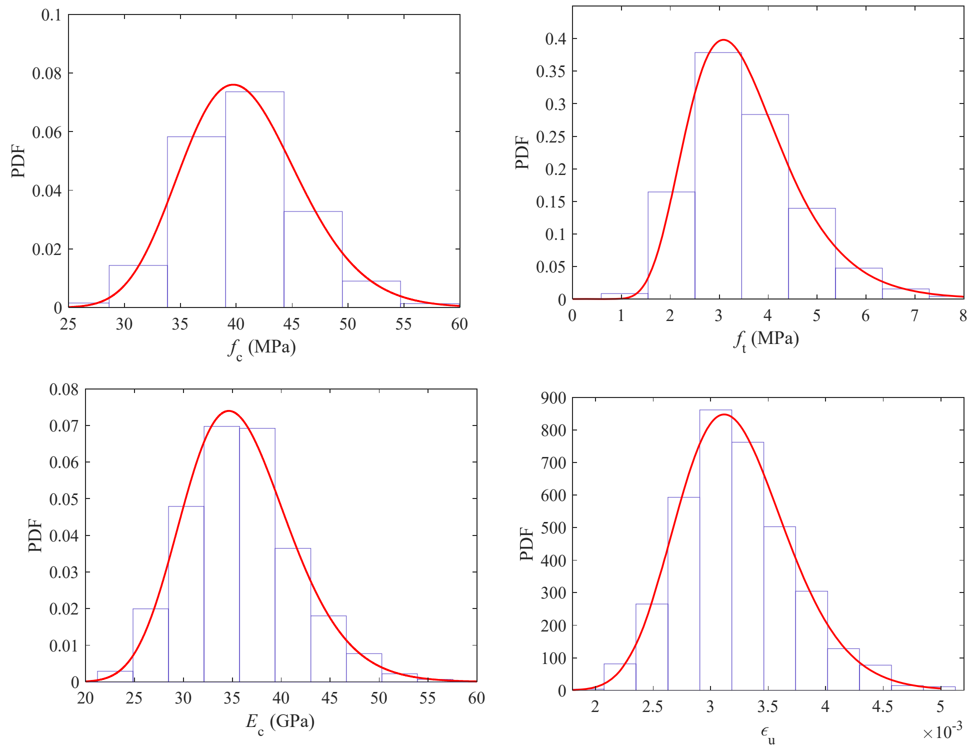
Variable	Distribution type	Mean	Coefficient of variation	Related to
$Y_{1,j}$	LN	1.0	0.06	Compression
$Y_{2,j}$	LN	1.0	0.30	Tension
$Y_{3,j}$	LN	1.0	0.15	E-modulus
$Y_{4,j}$	LN	1.0	0.15	Ultimate strain

2.1 JCSS Code

➢ The probabilistic model

E.g., C35. **Number of samples: 200**



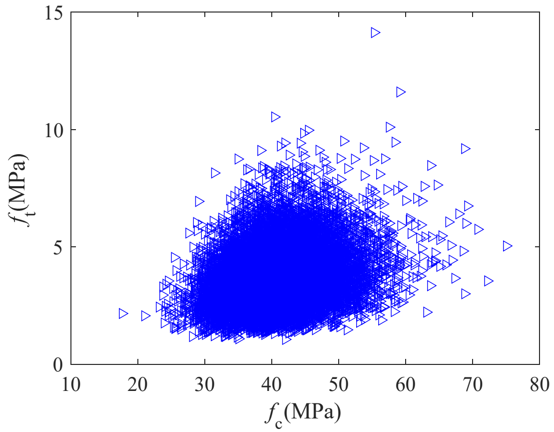


Variable	$f_{c,ij}$(MPa)	$f_{ct,ij}$	$E_{c,ij}$(GPa)	$\varepsilon_{u,ij}$
Mean	40.8	3.56	35.9	0.0033
Coefficient of variation	0.132	0.319	0.154	0.150

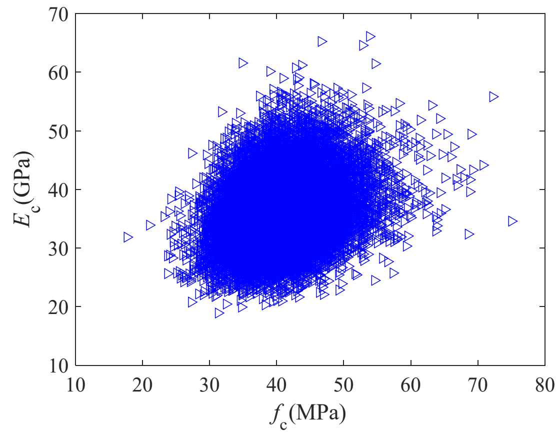
2.1 JCSS Code

➢ The probabilistic model

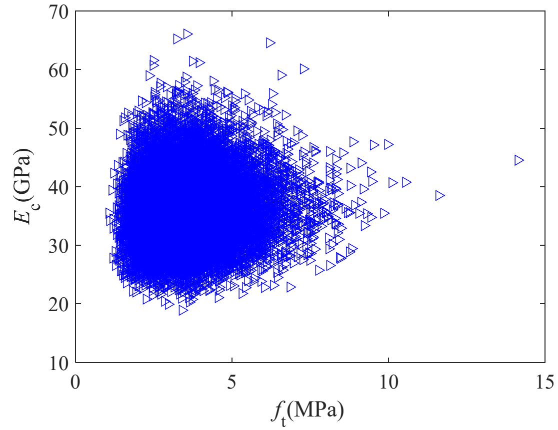
Number of samples: 20000



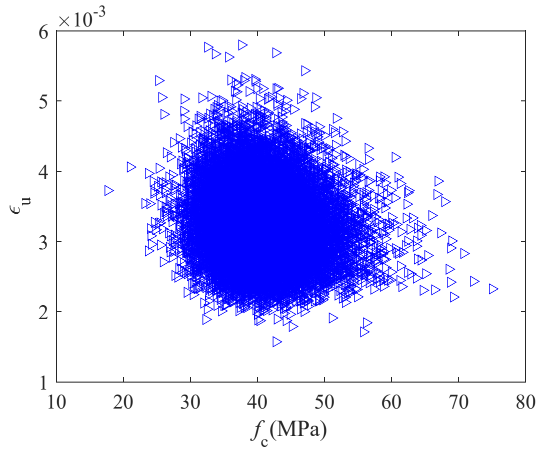
ρ=0.266



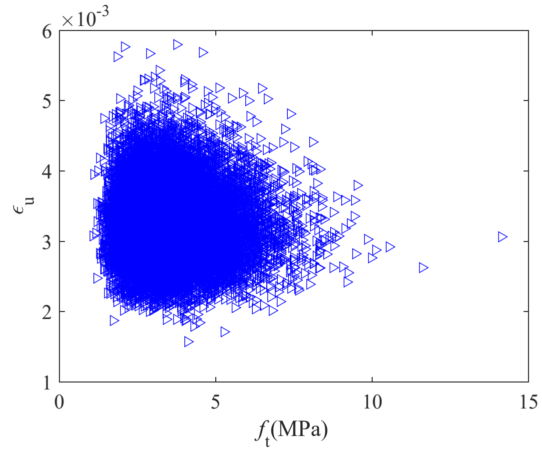
ρ=0.281



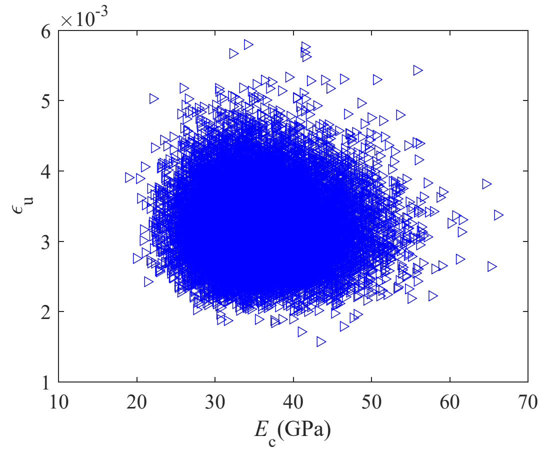
ρ=0.064



$\rho = -0.134$



$\rho = -0.037$



$\rho = -0.043$

Concrete properties

$$f_{cm} = f_{ck} + \Delta f;$$

$$\Delta f = 8\,\mathrm{MPa};$$

$$f_{ctm} = 0.3\,(f_{ck})^{2/3};$$

$$G_F = 73 \cdot f_{cm}^{0.18};$$

$$E_{ci} = E_{c0} \cdot \alpha_E \cdot \left(\frac{f_{ck} + \Delta f}{10}\right)^{1/3}$$

Where:

f_{cm}: a mean value of compressive strength;

f_{ck} : a specific characteristic compressive strength

f_{ctm}: a mean value of tensile strength;

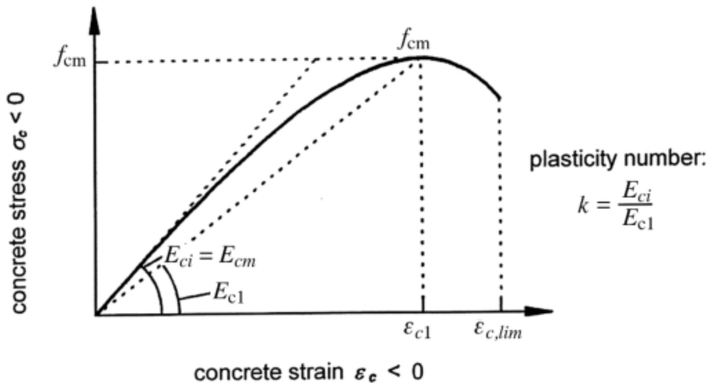
G_F : the fracture energy of concrete;

E_{c0} : $21.5 \times 10^3\,\mathrm{MPa}$

E_{ci} : the modulus of elasticity at the concrete age of 28 days;

Uniaxial compressive stress-strain curves

$$\frac{\sigma_c}{f_{cm}} = -\left(\frac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta}\right) \quad \text{for } |\varepsilon_c| < |\varepsilon_{c,\lim}|$$



Where:

$\eta = \varepsilon_c / \varepsilon_{c1}$;

$k = E_{ci} / E_{c1}$;

ε_{c1} : the strain at maximum compressive stress

E_{c1}: the secant modulus from the origin to the peak compressive stress

2.2 Fib Code

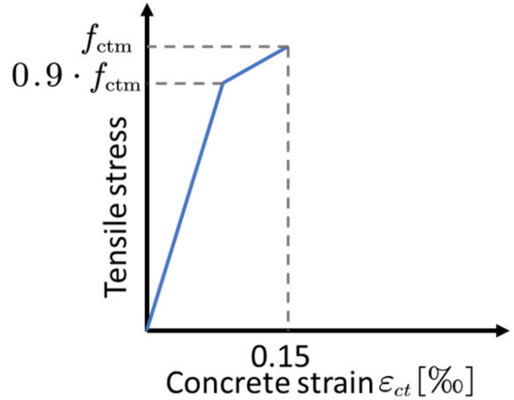
➢ Uniaxial tensile stress-strain curves

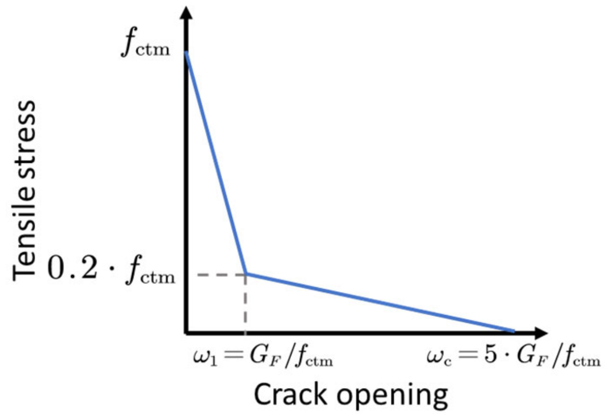
$$\sigma_{ct} = E_{ci} \cdot \varepsilon_{ct} \qquad \text{for } \sigma_{ct} \leq 0.9 \cdot f_{ctm}$$

$$\sigma_{ct} = f_{ctm} \cdot \left(1 - 0.1 \frac{0.00015 - \varepsilon_{ct}}{0.00015 - 0.9 \cdot f_{ctm}/E_{ci}}\right) \qquad \text{for } 0.9 \cdot f_{ctm} < \sigma_{ct} \leq f_{ctm}$$

$$\sigma_{ct} = f_{ctm} \cdot \left(1.0 - 0.8 \frac{w}{w_1}\right) \qquad \text{for } w \leq w_1$$

$$\sigma_{ct} = f_{ctm} \cdot \left(0.25 - 0.05 \frac{w}{w_1}\right) \qquad \text{for } w_1 < w \leq w_c$$





Where:

w : the crack opening in mm;

$w1$: G_F/f_{ctm} in mm when $\sigma_{ct} = 0.20 \cdot f_{ctm}$;

wc : $5 \cdot G_F/f_{ctm}$ in mm when $\sigma_{ct} = 0$.

2.2 Fib Codes

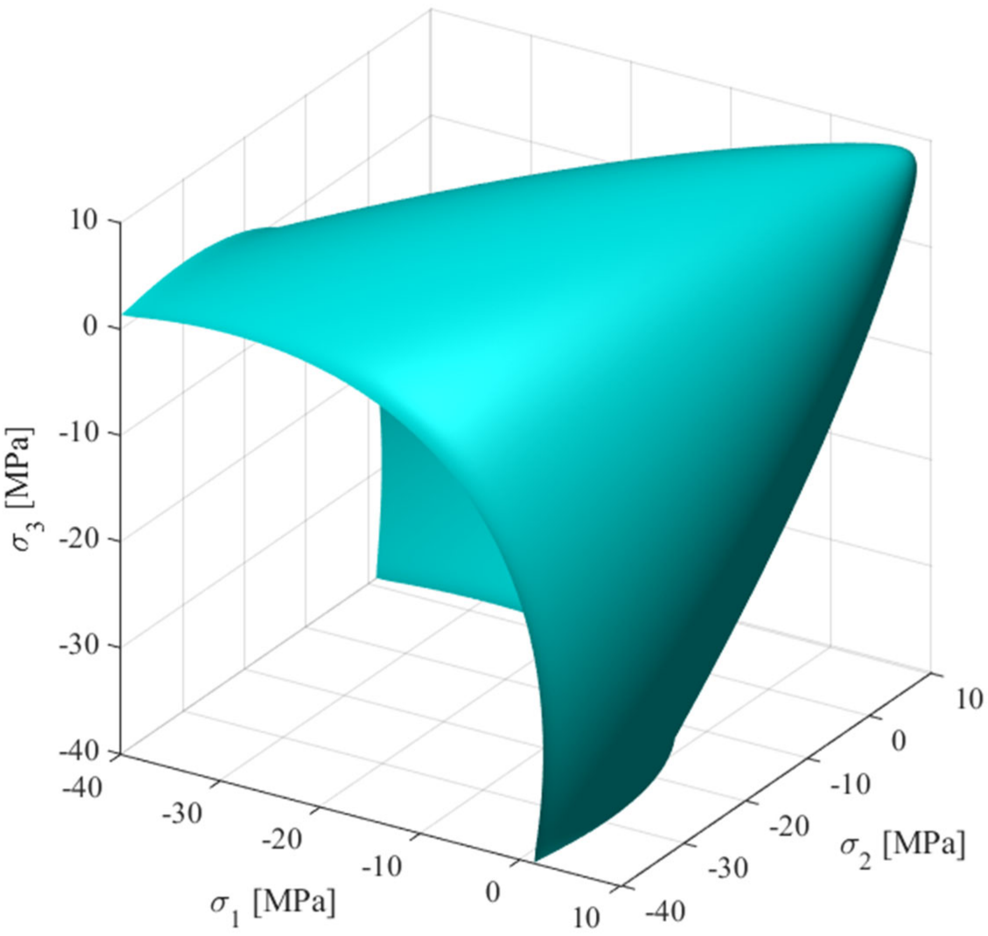
➢ **Failure criterion:**

$$\alpha\frac{J_2}{f_{\mathrm{cm}}^2}+\lambda\frac{\sqrt{J_2}}{f_{\mathrm{cm}}}+\beta\frac{I_1}{f_{\mathrm{cm}}}-1=0$$

where:

$$\lambda=c_1\cdot\cos\left[\frac{1}{3}\cdot\arccos\left(c_2\cdot\cos 3\theta\right)\right]$$

$$\cos 3\theta=\frac{3\sqrt{3}}{2}\cdot\frac{J_3}{J_2^{3/2}}$$



2.2 Fib Codes

➢ **Elastoplastic model:**

$$\sigma = E_0(\varepsilon - \varepsilon_\mathrm{p})$$

$$\dot{\varepsilon}_\mathrm{p} = \lambda \frac{\partial g}{\partial \sigma}$$

$$\lambda \geq 0, \quad f\lambda = 0, \quad f \leq 0$$

λ: the plastic multiplier

$$\dot{f} = \frac{\partial f}{\partial \sigma} \cdot \dot{\sigma} + \frac{\partial f}{\partial \alpha} \cdot \dot{\alpha} = 0$$

$$\dot{\alpha} = \lambda h(\sigma, \alpha)$$

$$\dot{\sigma} = \left[E_0 - \frac{E_0 \cdot \frac{\partial g}{\partial \sigma} \cdot \frac{\partial f^T}{\partial \sigma} \cdot E_0}{\frac{\partial f^T}{\partial \sigma} \cdot E_0 \cdot \frac{\partial g}{\partial \sigma} - \frac{\partial f^T}{\partial \alpha} \cdot h} \right]$$

➢ **Damage models:**

$$\sigma = E \cdot \varepsilon$$

$$\dot{E} = -\lambda' G$$

λ': the damage multiplier

$$\lambda' \geq 0, \quad F\lambda' = 0, \quad F \leq 0$$

$$\dot{F} = \frac{\partial F}{\partial \varepsilon} \cdot \dot{\varepsilon} + \frac{\partial F}{\partial \beta} \cdot \dot{\beta} = 0$$

$$\dot{\beta} = \lambda' h'(\varepsilon, \beta)$$

$$\dot{\sigma} = \left[E + \frac{1}{\frac{\partial F}{\partial \beta} \cdot h'} G \cdot \varepsilon \cdot \frac{\partial F^T}{\partial \varepsilon} \right] \cdot \dot{\varepsilon}$$

$$E = (1 - D)E_0, \quad E = -\dot{D}E_0, \quad G = E_0$$

2.2 Fib Codes

Development of strength with time:

$$f_{cm}(t) = \boxed{\beta_{cc}(t)} \cdot f_{cm}$$

a function to describe the strength development with time

$$E_{ci}(t) = \boxed{\beta_E(t)} \cdot E_{ci}$$

a coefficient which depends on the age of concrete

Creep and shrinkage:

$$\varepsilon_c(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(t) + \varepsilon_{cs}(t) + \varepsilon_{cT}(t)$$

ε_{ci}: the initial strain at loading;

ε_{cc}: the creep strain;

ε_{cs}: the shrinkage strain;

ε_{cT}: the thermal strain.

Temperature effects: $f_{cm}(T) = f_{cm}(1.06 - 0.003 \cdot T)$

Properties related to non-static loading:

Compressive strength:

$$\begin{cases} f_{c,imp,k}/f_{cm} = (\dot{\varepsilon}_c/\dot{\varepsilon}_{c0})^{0.014} & \text{for } \dot{\varepsilon}_c \leqslant 30s^{-1} \\ f_{c,imp,k}/f_{cm} = 0.012(\dot{\varepsilon}_c/\dot{\varepsilon}_{c0})^{1/3} & \text{for } \dot{\varepsilon}_c > 30s^{-1} \end{cases}$$

Tensile Strength:

$$\begin{cases} f_{ct,imp,k}/f_{ctm} = (\dot{\varepsilon}_{ct}/\dot{\varepsilon}_{ct0})^{0.018} & \text{for } \dot{\varepsilon}_{ct} \leqslant 10s^{-1} \\ f_{ct,imp,k}/f_{ctm} = 0.0062(\dot{\varepsilon}_{ct}/\dot{\varepsilon}_{ct0})^{1/3} & \text{for } \dot{\varepsilon}_{ct} > 10s^{-1} \end{cases}$$

2.3 Eurocode2 Codes

➢ Concrete properties

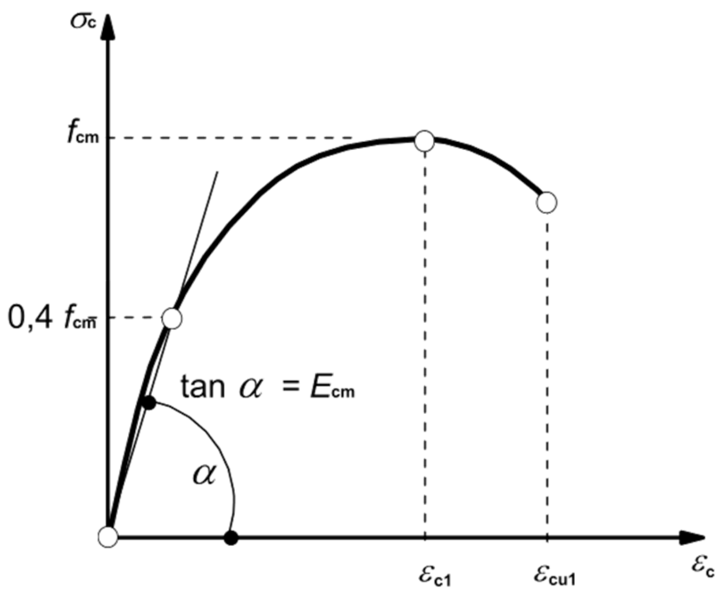
$$f_{ck}(t) = f_{cm}(t) - 8(\mathrm{MPa}) \quad \text{for } 3 < \mathrm{t} < 28\,\mathrm{days}$$

$$f_{ck}(t) = f_{ck} \quad \text{for t} \geq 28\,\mathrm{days}$$

$$f_{ct} = 0.9 f_{ct,sp}$$

➢ Uniaxial compressive stress-strain curves

$$\frac{\sigma_c}{f_{cm}} = -\left(\frac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta}\right) \quad \text{for } |\varepsilon_c| < |\varepsilon_{cu1}|$$



Where:

f_{cm}: a mean value of compressive strength;

f_{ck} : a specific characteristic compressive strength;

f_{ctm}: a mean value of tensile strength;

G_F : the fracture energy of concrete;

E_{c0} : 21.5×10^3 MPa

E_{ci} : the modulus of elasticity at the concrete age of 28 days;

2.3 Eurocode2 Codes

Development of strength with time：

$$f_{cm}(t) = \boxed{\beta_{cc}(t)} \cdot f_{cm} \qquad E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0.3} E_{cm}$$

a function to describe the strength development with time

Creep and shrinkage: $\varepsilon_c(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(\infty, t_0) + \varepsilon_{cs}$; $\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$

Where:

ε_c:the Total strain; $\varepsilon_{ci(t_0)}$:the initial strain at loading;

$\varepsilon_{cc}(\infty, t_0)$:the creep strain; ε_{cs}:the shrinkage strain;

ε_{cd}:the drying shrinkage strain; ε_{ca}:the autogenous shrinkage strain.

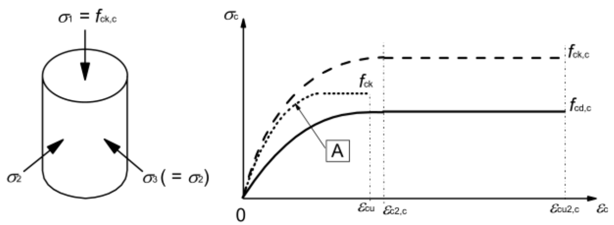
Confined concrete：

for $\sigma_2 \leqslant 0.05 f_{ck}$:

$$f_{ck,c} = f_{ck}(1.0 + 5.0\sigma_2/f_{ck})$$

for $\sigma_2 > 0.05 f_{ck}$:

$$f_{ck,c} = f_{ck}(1.125 + 2.5\sigma_2/f_{ck})$$



2.4 Chinese Codes

➢ Concrete properties

$$f_{\mathrm{cm}} = f_{\mathrm{ck}} / (1 - 1.645\delta_c);$$

$$f_{\mathrm{tm}} = f_{\mathrm{tk}} / (1 - 1.645\delta_c);$$

$$E_{\mathrm{c}} = \frac{10^5}{2.2 + \frac{34.7}{f_{\mathrm{cu,k}}}} \ (\mathrm{MPa})$$

$$\varepsilon_{\mathrm{t,r}} = f_{\mathrm{t,r}}^{0.54} \times 65 \times 10^{-6}$$

$$\alpha_{\mathrm{t}} = 0.312 f_{\mathrm{t,r}}^2$$

$$\varepsilon_{\mathrm{c,r}} = \left(700 + 172\sqrt{f_{\mathrm{c}}}\right) \times 10^{-6}$$

$$\alpha_{\mathrm{c}} = 0.157 f_{\mathrm{c}}^{0.785} - 0.905$$

$$\frac{\varepsilon_{\mathrm{cu}}}{\varepsilon_{c,\mathrm{r}}} = \frac{1}{2\alpha_{\mathrm{c}}}\left(1 + 2\alpha_{\mathrm{c}} + \sqrt{1 + 4\alpha_{\mathrm{c}}}\right)$$

Where:

$f_{\mathrm{cm/tm}}$: a mean value of strength;

$f_{\mathrm{ck/tk}}$: a specific characteristic strength;

δ_c : coefficient of variation of concrete strength;

$f_{\mathrm{cu,k}}$: standard values for compressive strength of cubes;

E_{c} : modulus of elasticity;

$\varepsilon_{\mathrm{t,r/c,r}}$: peak tensile/compression strain;

$\alpha_{\mathrm{t/c}}$:tensile/compression damage curve parameters.

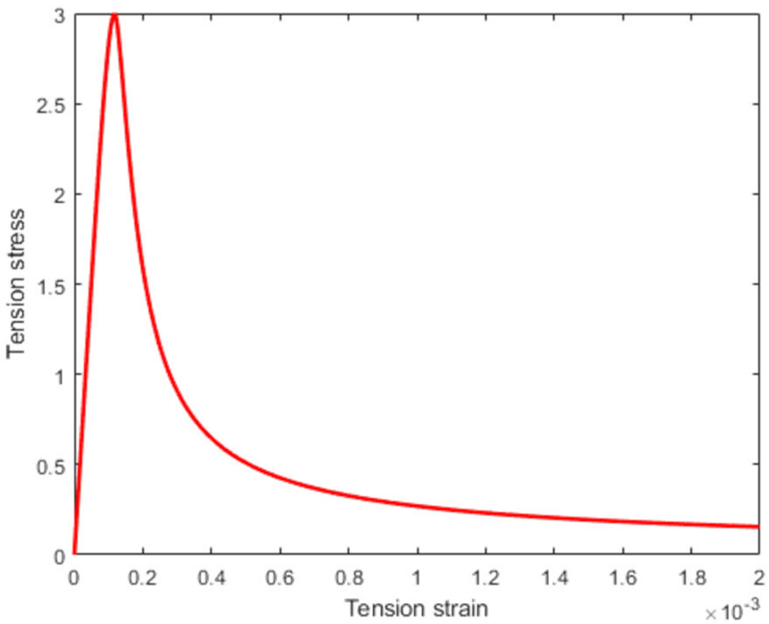
2.4 Chinese Codes

Tension Damage:

$$\sigma=(1-d_t)E_c\varepsilon$$

$$d_t=\begin{cases}1-\rho_t[1.2-0.2x^5] & x\leqslant 1\\ 1-\dfrac{\rho_t}{\alpha_t(x-1)^{1.7}+x} & x>1\end{cases}$$

$$x=\frac{\varepsilon}{\varepsilon_{t,r}};\ \rho_t=\frac{f_{t,r}}{E_c\varepsilon_{t,r}}$$

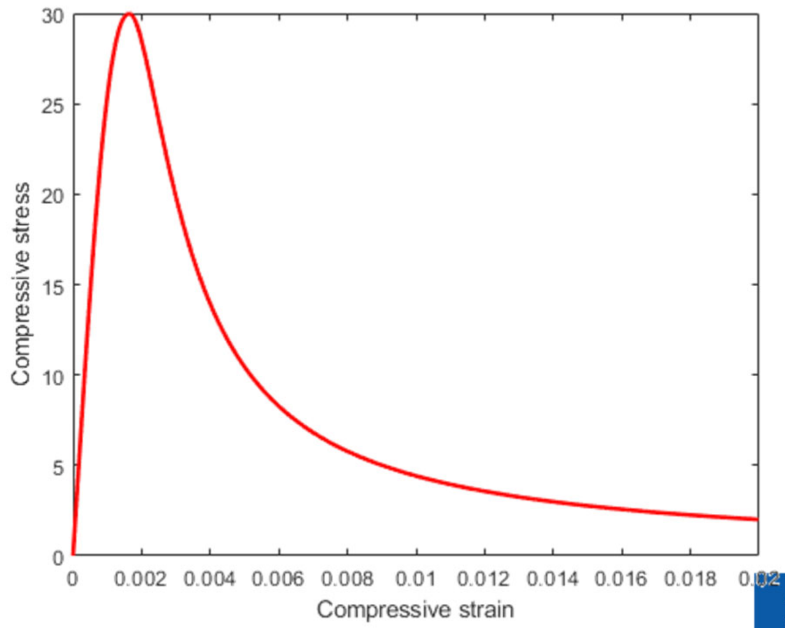


Compression damage:

$$\sigma=(1-d_c)E_c\varepsilon$$

$$d_c=\begin{cases}1-\dfrac{\rho_c n}{n+1+x^n} & x\leqslant 1\\ 1-\dfrac{\rho_c}{\alpha_c(x-1)^2+x} & x>1\end{cases}$$

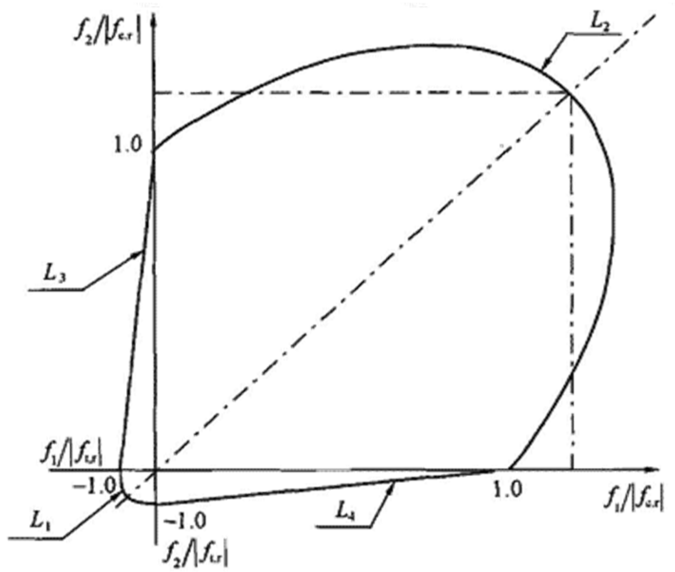
$$\rho_c=\frac{f_{c,r}}{E_c\varepsilon_{c,r}};\ x=\frac{\varepsilon}{\varepsilon_{c,r}};\ n=\frac{E_c\varepsilon_{c,r}}{E_c\varepsilon_{c,r}-f_{c,r}}$$



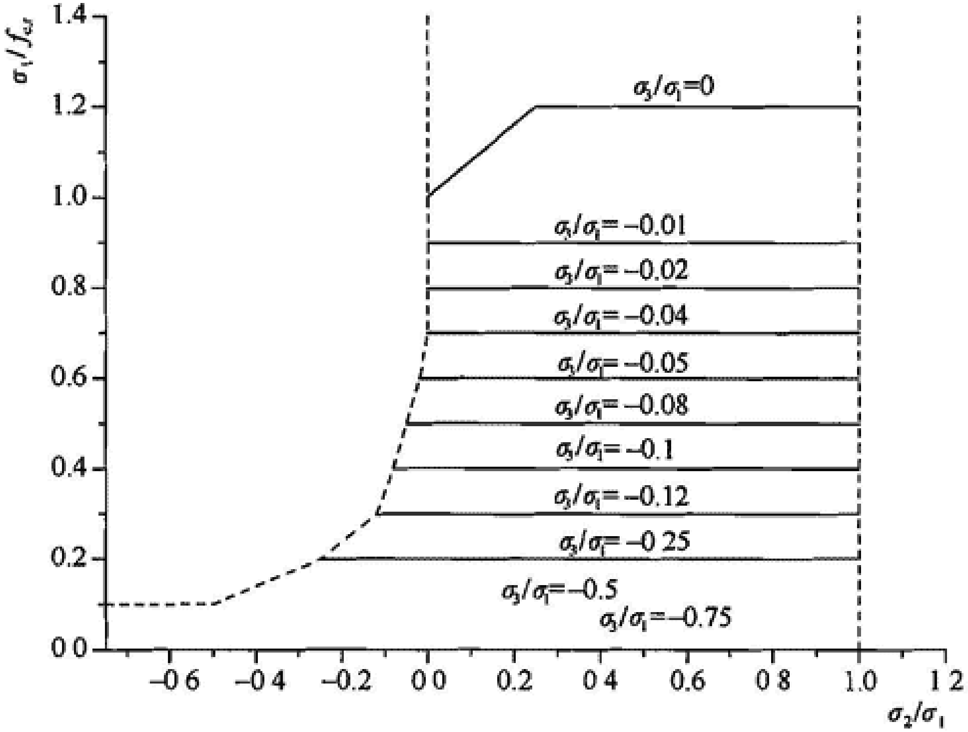
2.4 Chinese Codes

- **Biaxial stress state:**

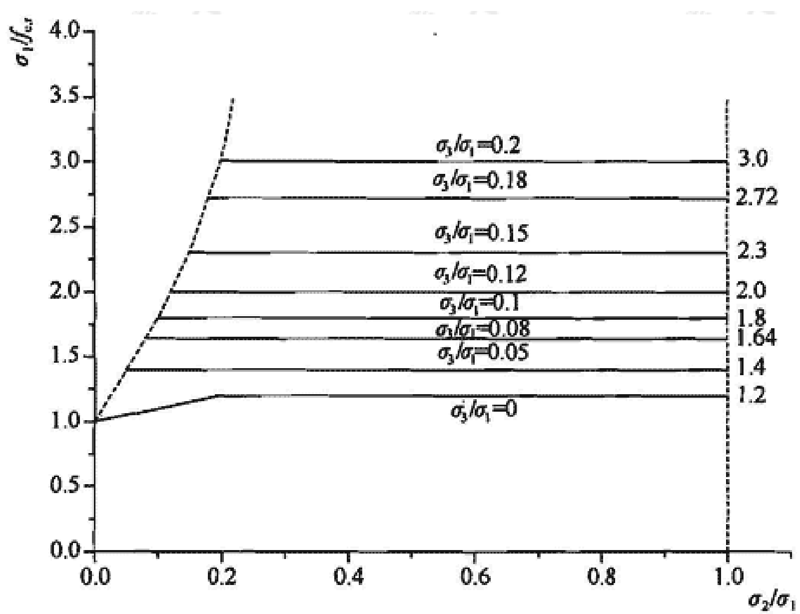
$$
\begin{cases}
L_1:\ f_1^2 + f_2^2 - 2\nu f_1 f_2 = (f_{\mathrm{t,r}})^2 \\
L_2:\ \sqrt{f_1^2 + f_2^2 - f_1 f_2} - \alpha_{\mathrm{s}}(f_1 + f_2) = (1 - \alpha_s)\,|f_{c,r}| \\
L_3:\ \dfrac{f_2}{|f_{\mathrm{t,r}}|} - \dfrac{f_1}{|f_{\mathrm{t,r}}|} = 1 \\
L_4:\ \dfrac{f_1}{|f_{\mathrm{t,r}}|} - \dfrac{f_2}{|f_{\mathrm{c,r}}|} = 1
\end{cases}
$$



- **Triaxial stress state:**



Triaxial compressive strength of concrete under triaxial tensile-compressive stress state



Triaxial compressive strength of concrete under triaxial compression

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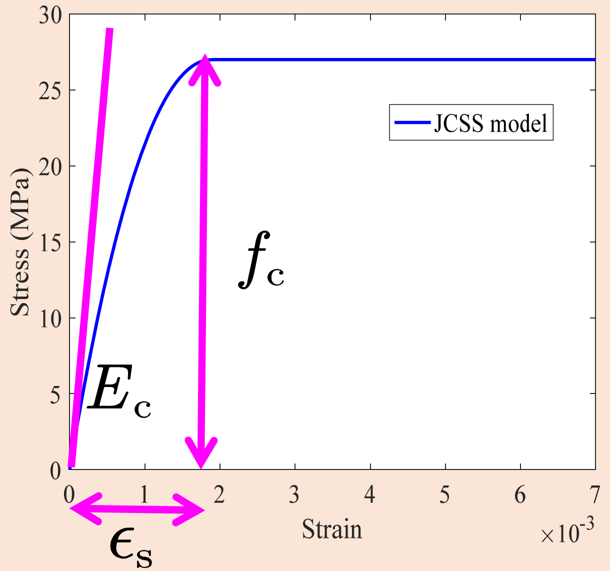
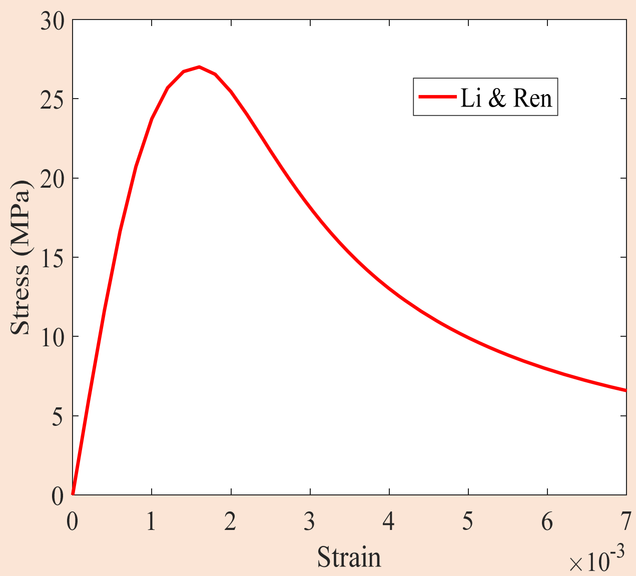
Contents

3.1 Uniaxial compression

➢ Consideration of descending part

$$\sigma = \begin{cases} f_c\left[1-\left(1-\dfrac{\epsilon}{\epsilon_s}\right)^k\right], & \text{for } 0<\epsilon<\epsilon_s \\ f_c, & \text{for } \epsilon \geqslant \epsilon_s \end{cases}$$





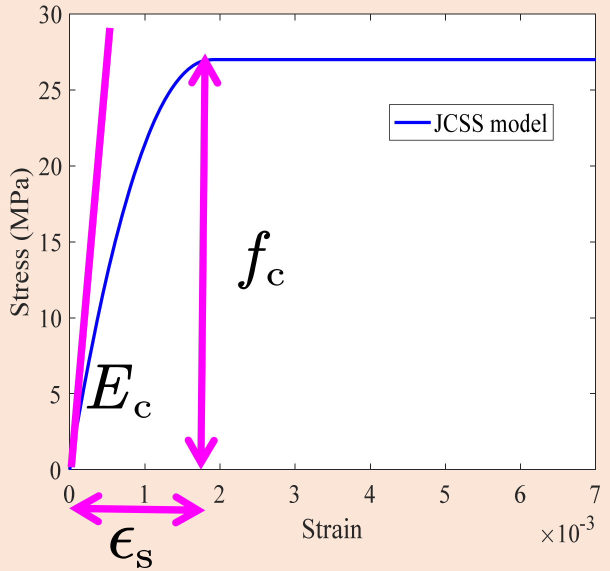
Sectional analysis and design

Nonlinear analysis and numerical simulation

3.1 Uniaxial compression

➢ Involvement of damage concept

$$\sigma = \begin{cases} f_c\left[1-\left(1-\dfrac{\epsilon}{\epsilon_s}\right)^k\right], & \text{for } 0<\epsilon<\epsilon_s \\ f_c, & \text{for } \epsilon \geqslant \epsilon_s \end{cases}$$



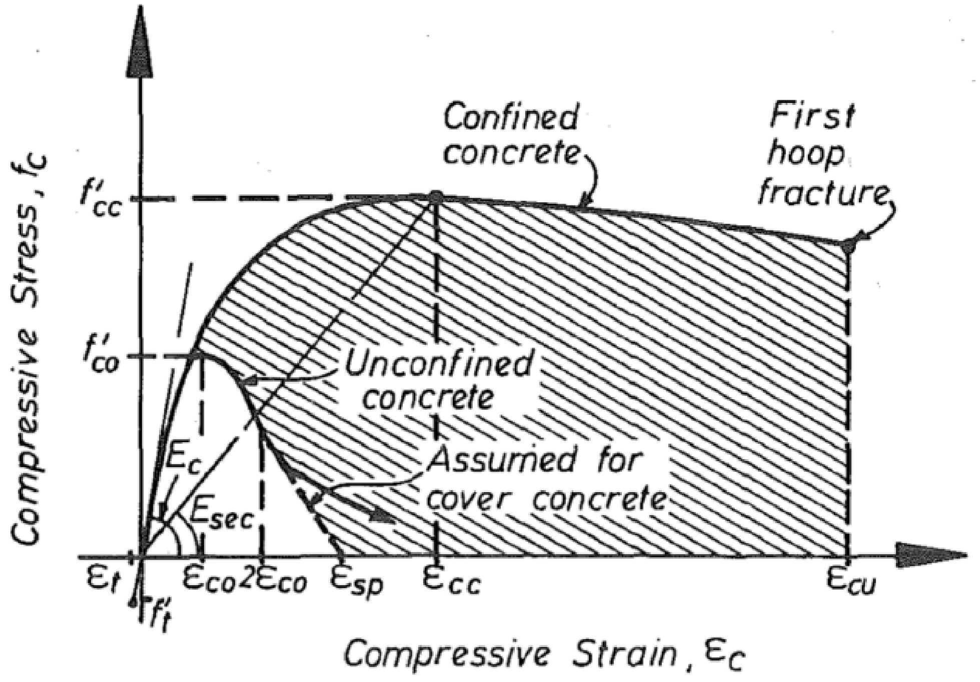
$$\sigma = f(\varepsilon)$$

$$\sigma = (1-d)E_c\epsilon$$

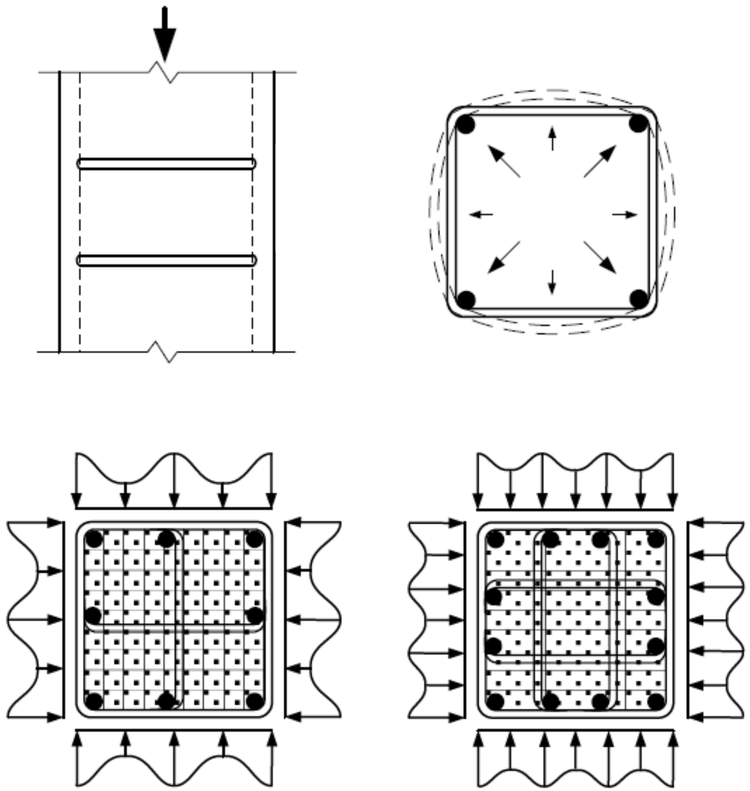
Damage index/variable

Sectional analysis and design

- Involvement of (passive) confinement effect

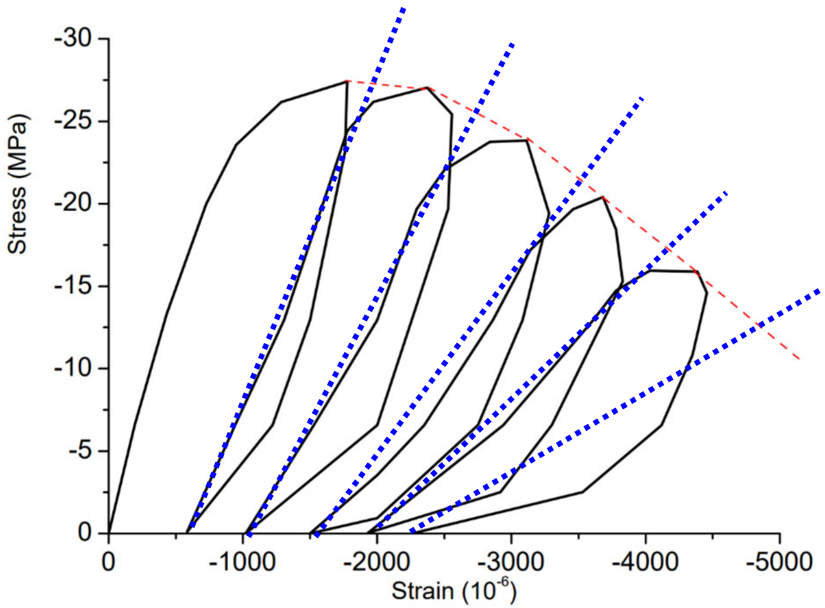


Stress-strain models

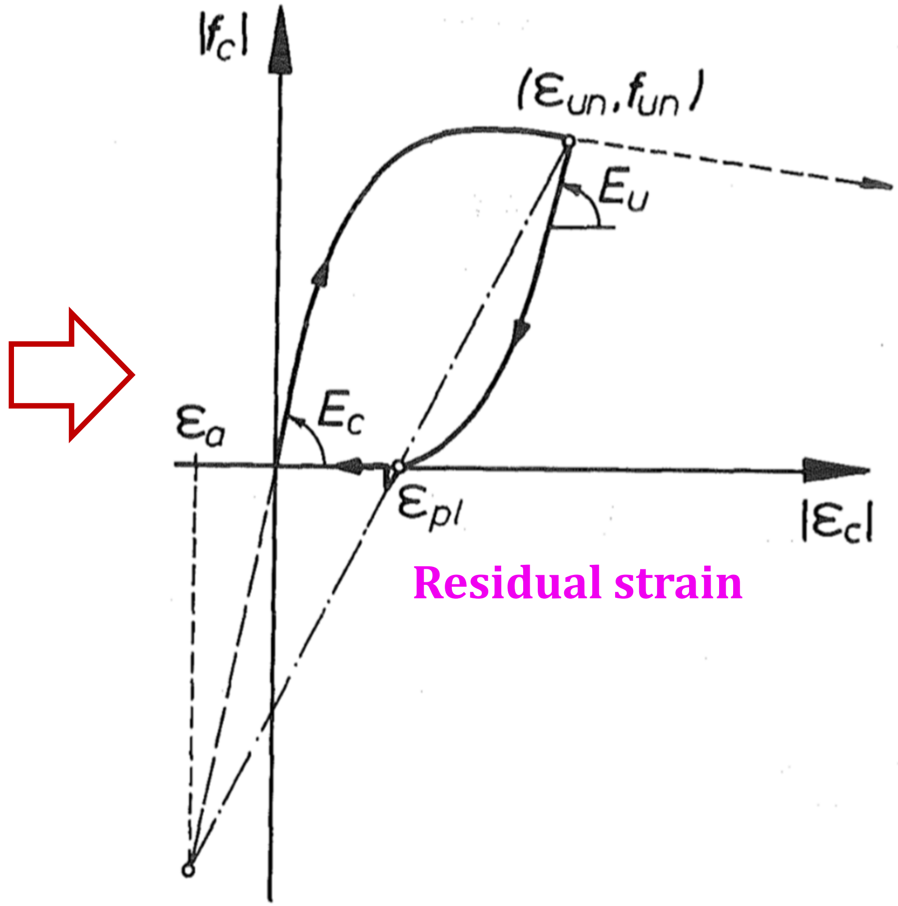


Confinement effect

- Residual strain

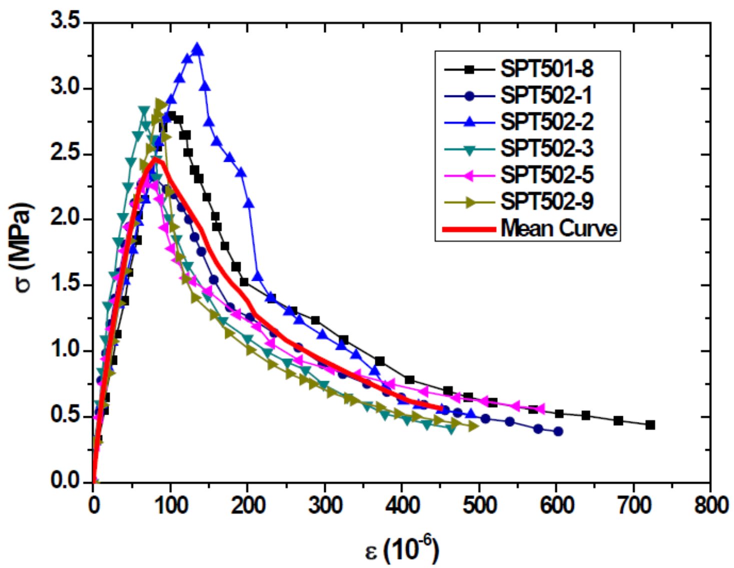


Repeated loading curves

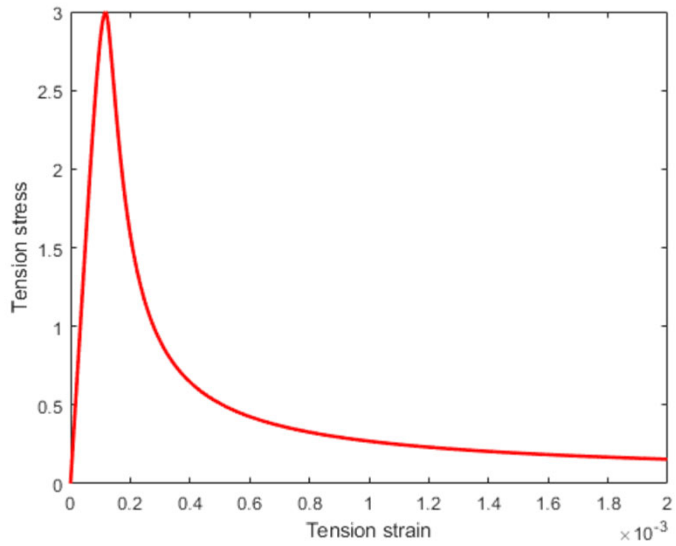


3.2 Uniaxial tension

➢ Continuum models for tension



Experimental results



$$\sigma = f(\varepsilon)$$

or

$$\sigma = (1 - d_t) E_c \varepsilon$$

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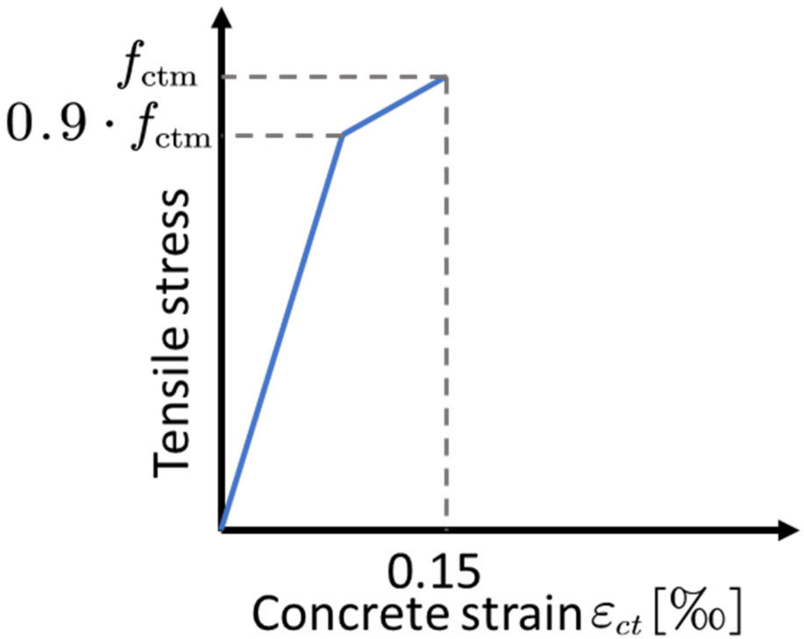
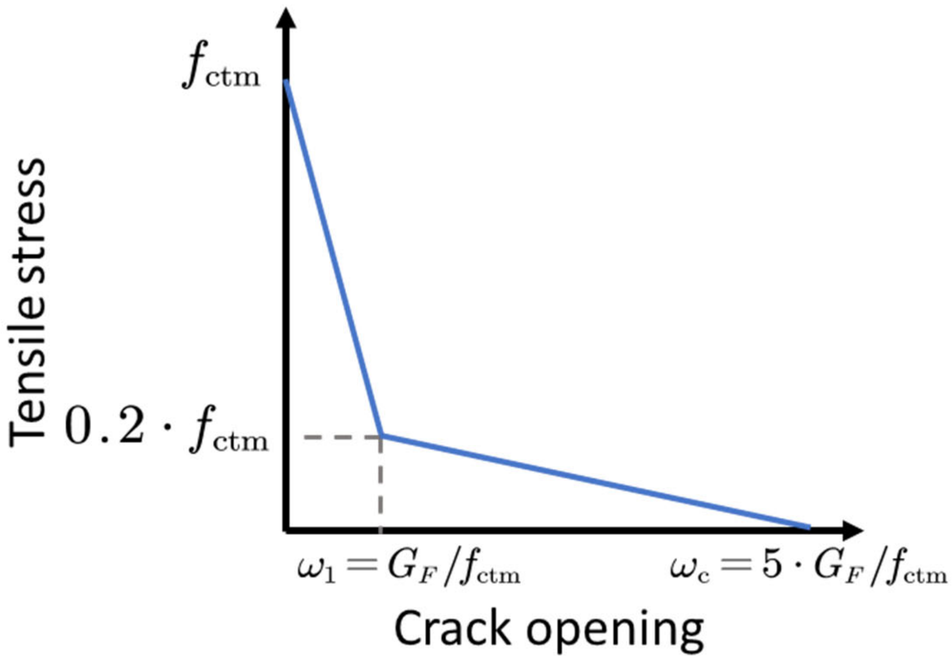
➢ Fracture model under tension

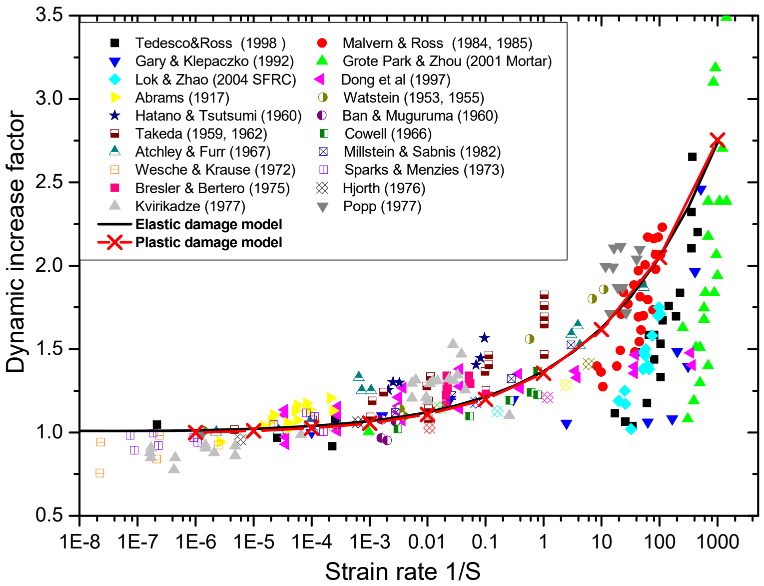




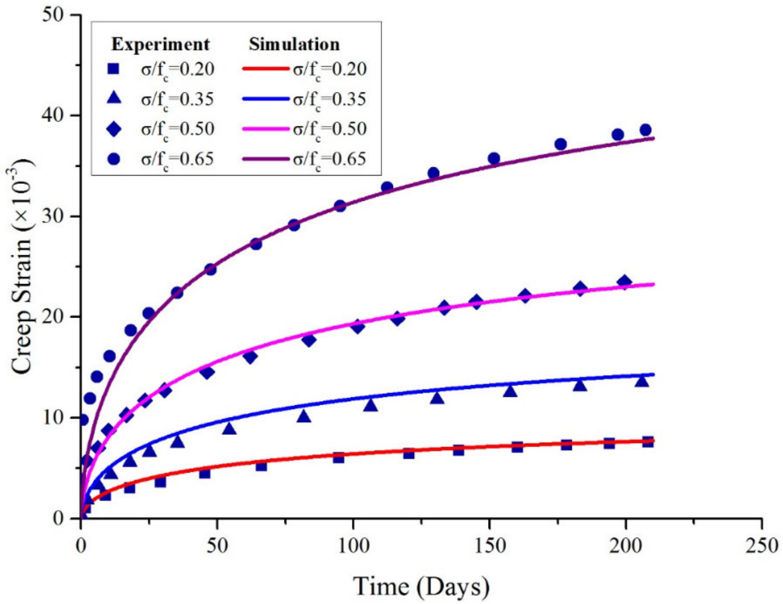
Randomness of fracture energy?

3.3 Other properties

➢ Time dependent behaviors



Dynamic increase of strength



Creep deformations

3.4 Multiaxial behaviors

➢ Biaxial envelop

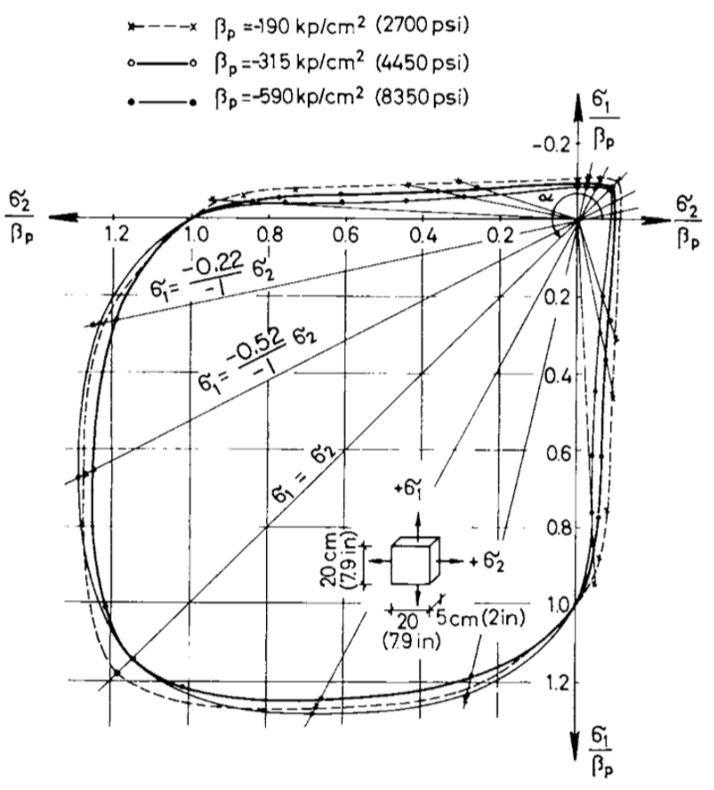
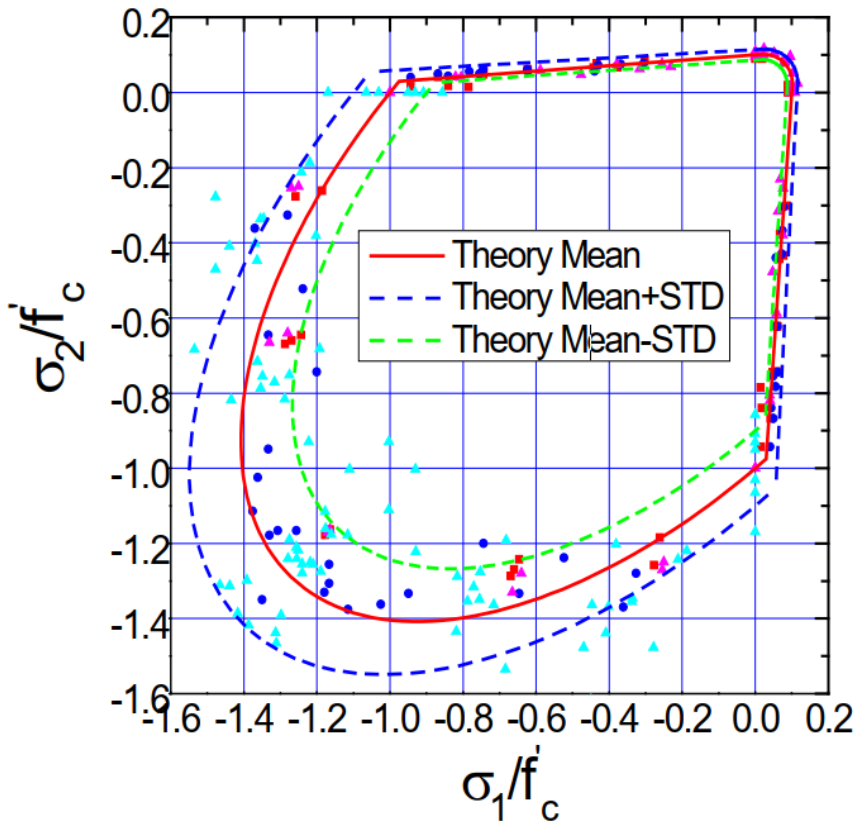


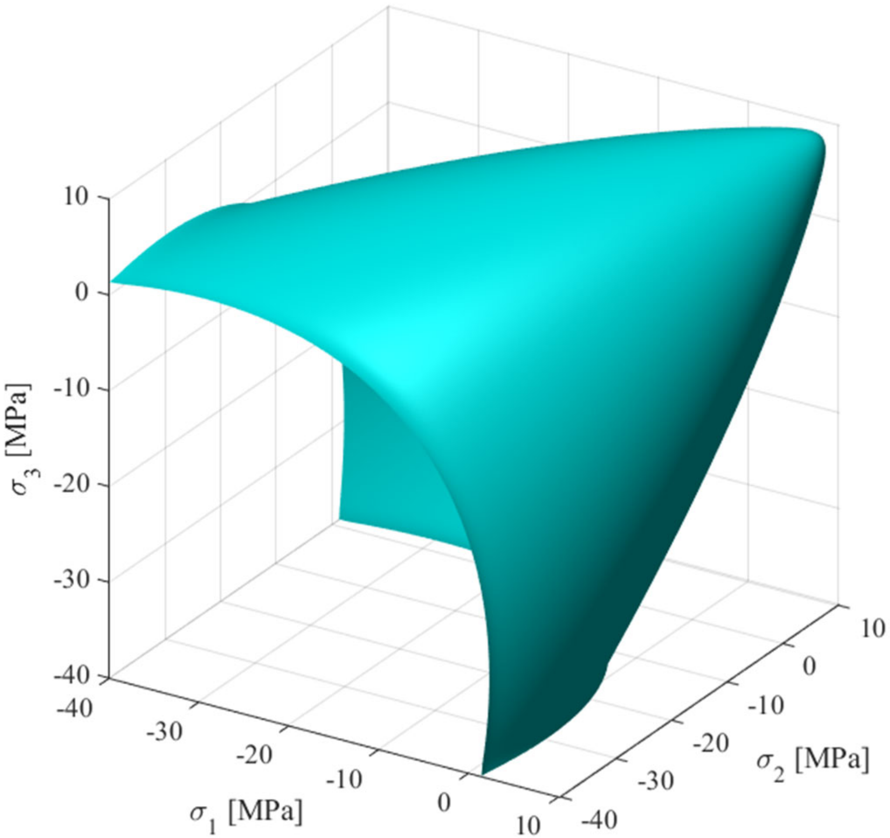
Fig. 6—Biaxial strength of concrete; results of experimental investigation



Randomness of biaxial envelop?

3.4 Multiaxial behaviors

➢ Full multiaxial criterion



$$F(\bar{\sigma},\kappa)=\left(\alpha \bar{I}_1+\sqrt{3\bar{J}_2}+\beta\langle\bar{\sigma}_{\max}\rangle\right)-(1-\alpha)\bar{c}\leqslant 0$$

3.4 Multiaxial behaviors

➢ Plasticity model

$$\boldsymbol{\sigma} = \mathbf{E}_0 : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right)$$

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \frac{\partial \tilde{f}(\boldsymbol{\sigma}, \boldsymbol{q})}{\partial \boldsymbol{\sigma}}$$

$$\begin{cases} \dot{\lambda} = \dfrac{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbb{E}_0 : \dot{\boldsymbol{\varepsilon}}}{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbb{E}_0 : \dfrac{\partial \tilde{f}}{\partial \boldsymbol{\sigma}} + \dfrac{\partial f}{\partial \boldsymbol{q}} \cdot \boldsymbol{H} \cdot \dfrac{\partial \tilde{f}}{\partial \boldsymbol{q}}} \\ \dot{\boldsymbol{\varepsilon}}^p = \left(\dfrac{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbb{E}_0 : \dfrac{\partial \tilde{f}}{\partial \boldsymbol{\sigma}}}{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbb{E}_0 : \dfrac{\partial \tilde{f}}{\partial \boldsymbol{\sigma}} + \dfrac{\partial f}{\partial \boldsymbol{q}} \cdot \boldsymbol{H} \cdot \dfrac{\partial \tilde{f}}{\partial \boldsymbol{q}}} \right) \dot{\boldsymbol{\varepsilon}} \end{cases}$$

➢ Damage model

$$\begin{aligned} \boldsymbol{\sigma} &= (\mathbb{I} - \mathbb{D}) : \bar{\boldsymbol{\sigma}} \\ &= (\mathbb{I} - \mathbb{D}) : \mathbf{E}_0 : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{\mathrm{p}}) \end{aligned}$$

$$\mathbb{D} = d^+ \mathbb{P}^+ + d^- \mathbb{P}^-$$

Damage (tensor)

$$\boldsymbol{\epsilon}^{\mathrm{p}} = \mathbf{E}_0^{-1} : \mathbf{F}(\mathbb{D}) : \mathbf{E}_0 : \boldsymbol{\epsilon}$$

Plastic strain

3.5 Probabilistic model

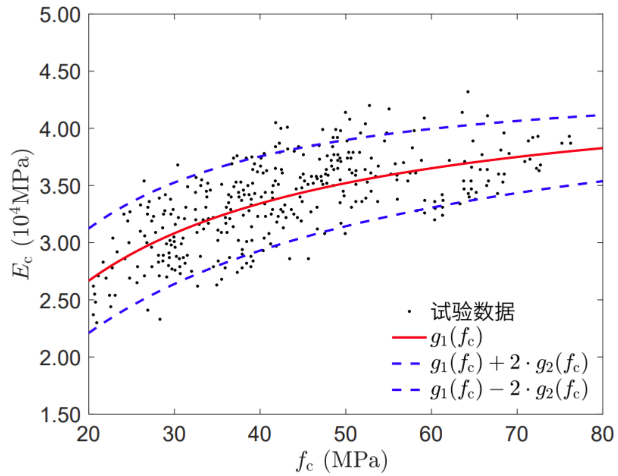
➢ Parameters of Constitutive model

☐ JCSS method

$$Y = \zeta \cdot g_1(X)$$

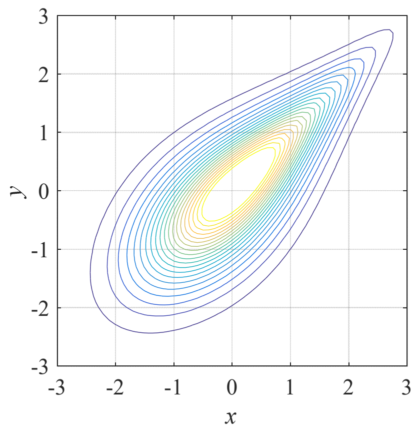
☐ Random function model

$$Y = g_1(X) + \zeta \cdot g_2(X)$$

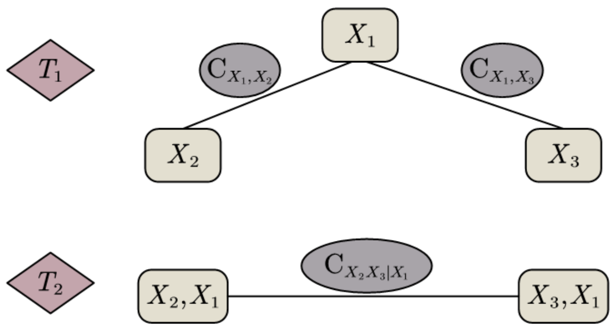


☐ Copula function

$$\begin{aligned} f(x_1,x_2,x_3) = {} & f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3) \\ & \cdot c_{12}\{F_1(x_1),F_2(x_2)\} \\ & \cdot c_{13}\{F_1(x_1),F_3(x_3)\} \\ & \cdot c_{23|1}\{F_{2|1}(x_2\mid x_1),F_{3|1}(x_3\mid x_1)\} \end{aligned}$$



$\mathbf{X} = [X_1, X_2, X_3]$



Vine copula

➢ Spatial variation

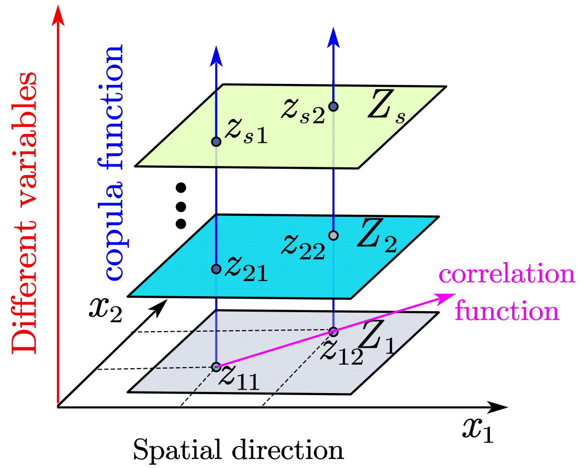
◻ JCSS method

$$\rho\left(U_{ij},U_{kj}\right)=\rho+\left(1-\rho\right)\exp\left[-\frac{\left(r_{ij}-r_{kj}\right)^{2}}{d_{c}^{2}}\right]$$

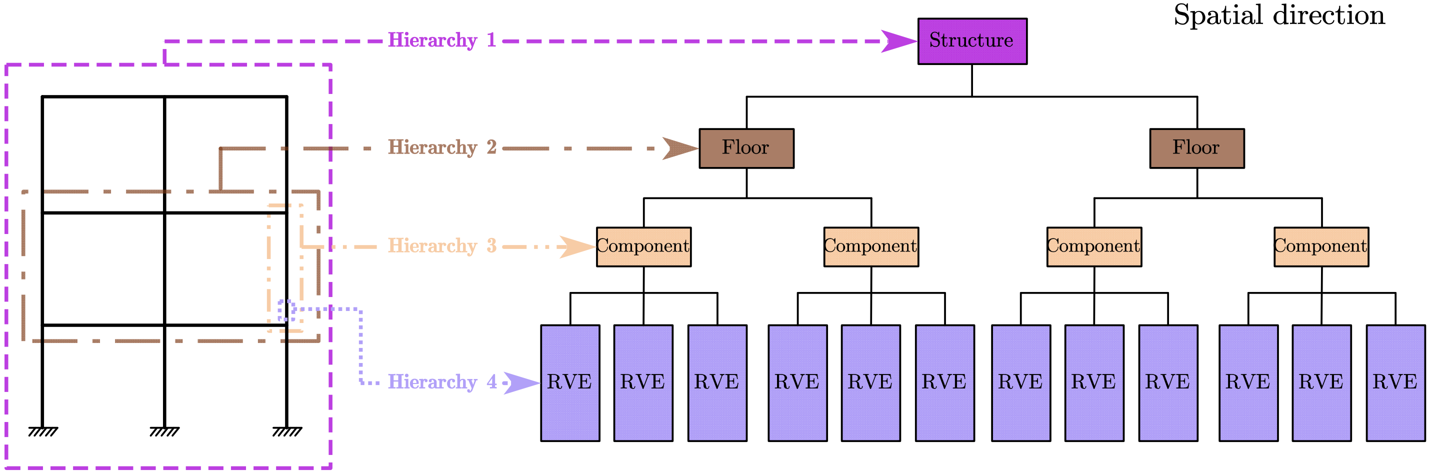
$$\Phi_{ijk}=f\left(Y_{i},Y_{ij},Y_{ijk}\right)$$

◻ Synthesizing the spatial correlation function and the multi-variate copula function

$$f_{Z_{21}Z_{22}Z_{11}Z_{12}}(z_{21},z_{22},z_{11},z_{12})$$
$$=f_{Z_{21}|Z_{11}}(z_{21}|z_{11})\cdot f_{Z_{22}|Z_{12}}(z_{22}|z_{12})f_{Z_{11}Z_{12}}(z_{11},z_{12})$$



◻ Variation in structures



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Contents

4.1 Randomness of Concrete

➢ Uncertainty quantification of constitutive parameters

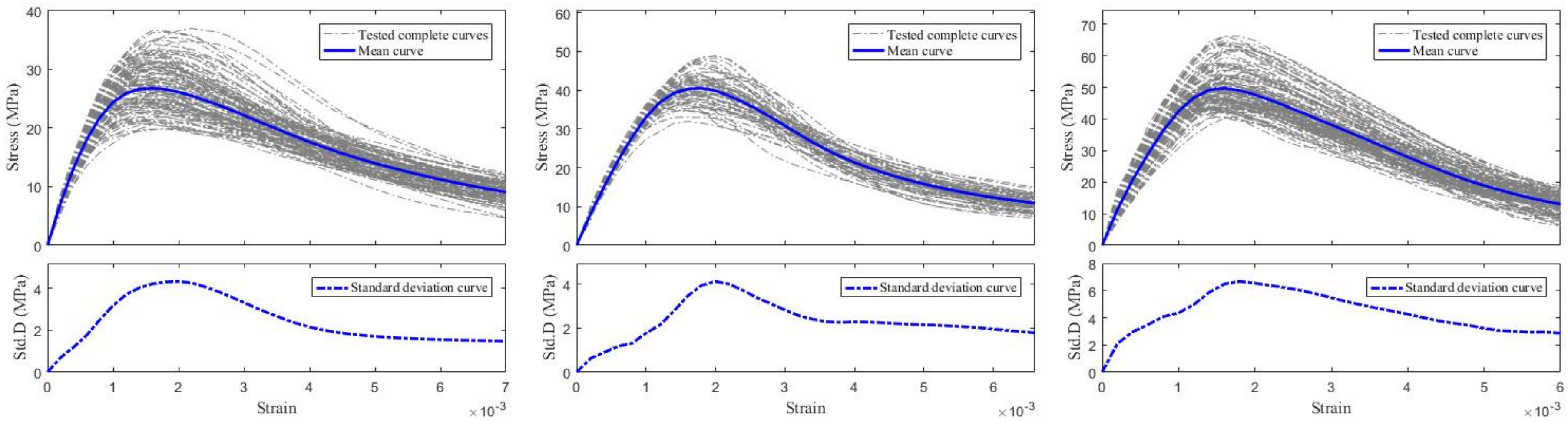
$$\sigma = (1 - d_c) E_c \varepsilon$$

$$d_c = \begin{cases} 1 - \dfrac{\rho_c n}{n - 1 + x^n} & x \leqslant 1 \\ 1 - \dfrac{\rho_c}{\alpha_c (x-1)^2 + x} & x > 1 \end{cases}$$

$$\rho_c = f_c/(E_c \varepsilon_c),\ n = E_c \varepsilon_c/(E_c \varepsilon_c - f_c),\ x = \varepsilon/\varepsilon_c$$

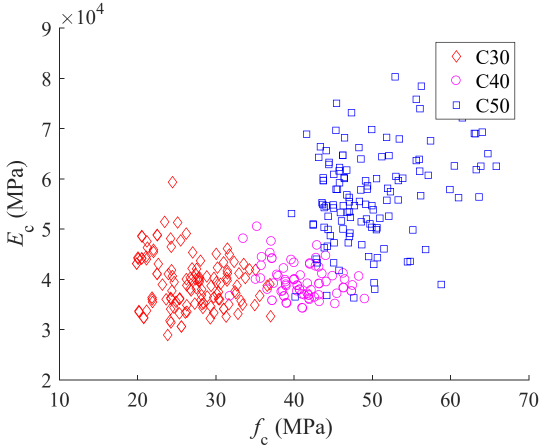
4 parameters

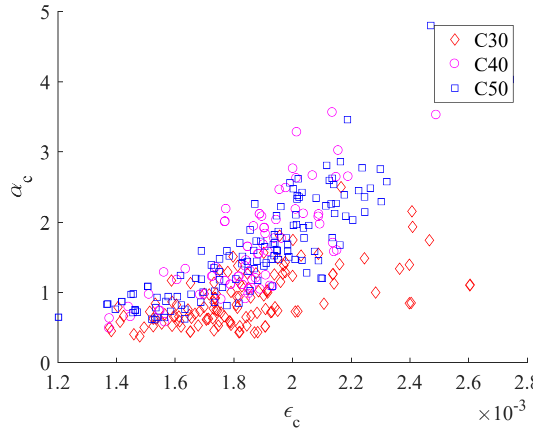
- Strength f_c
- Modulus of elasticity E_c
- Peak strain ε_c
- Descendent parameter α_c

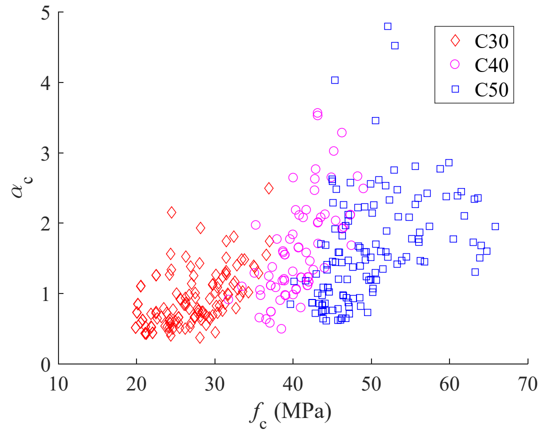


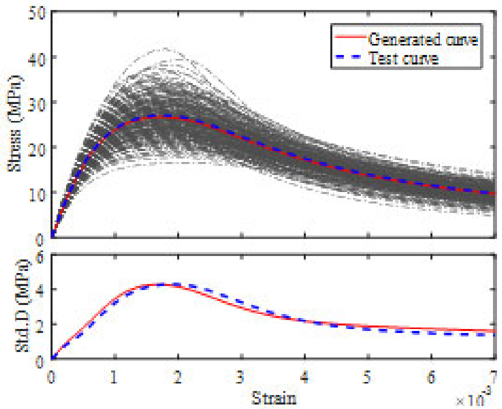
C30 C40 C50

4.1 Randomness of Concrete

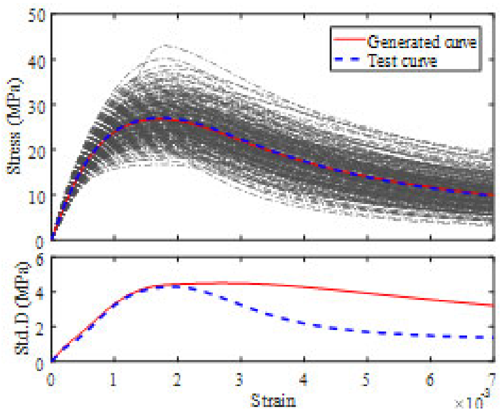




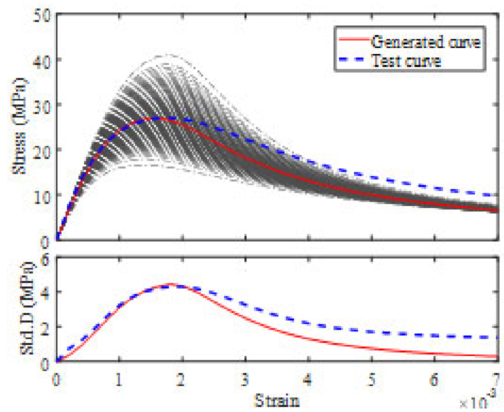




Copula dependent parameters (true)



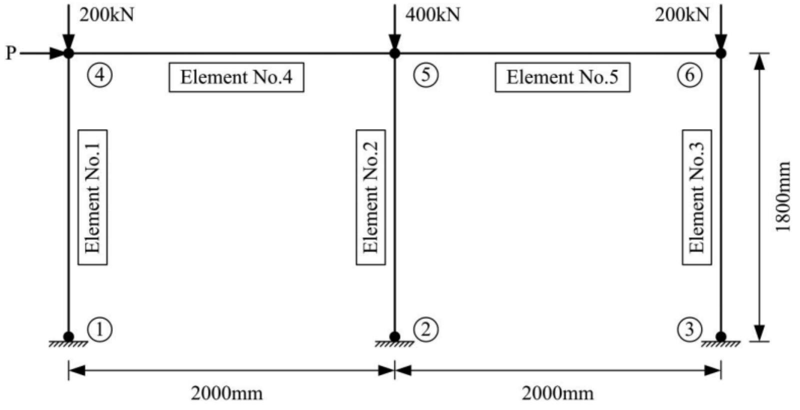
Independent parameters

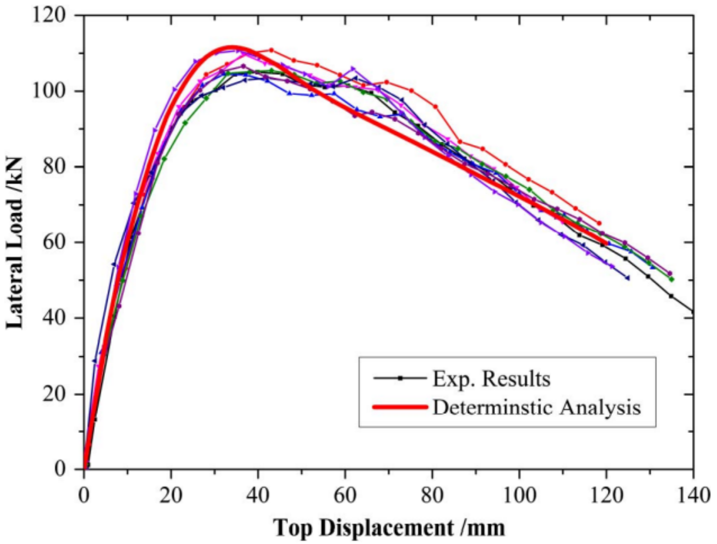


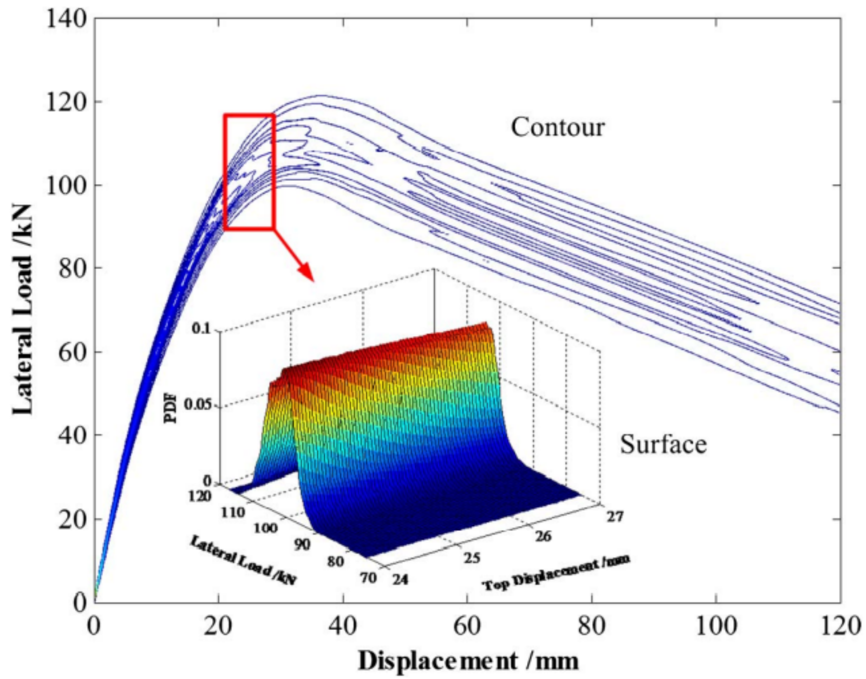
Perfect-correlated parameters

Chen JB, Tao JJ, Ren XD, Li J. China Civil Engineering Journal, 2020, 53(7): 52-63.
Tao JJ, Chen JB, Ren XD. Journal of Structural Engineering (ASCE), 2020, 146(9): 04020194.

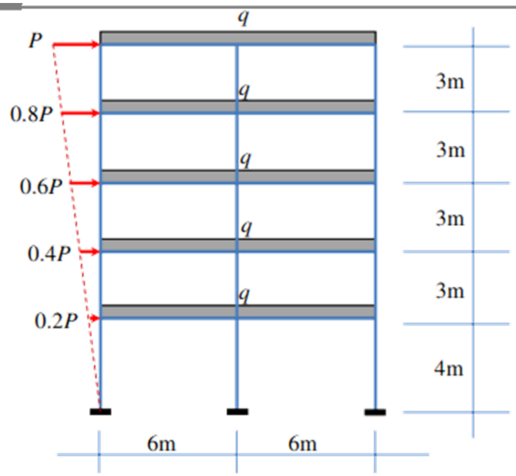
4.2 Stochastic Response Analysis

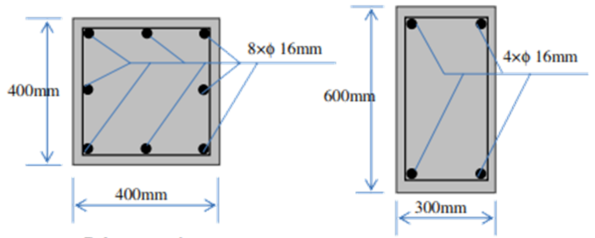




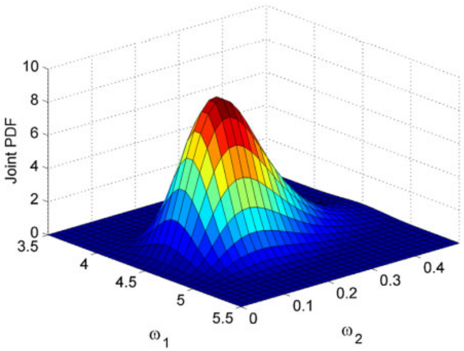


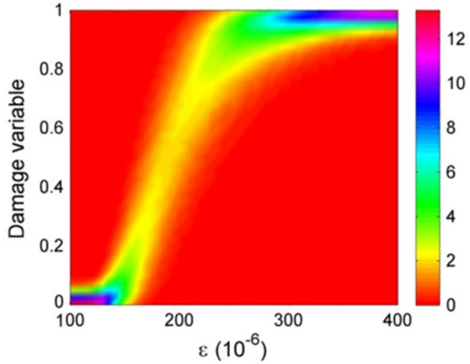
Stochastic Response Analysis

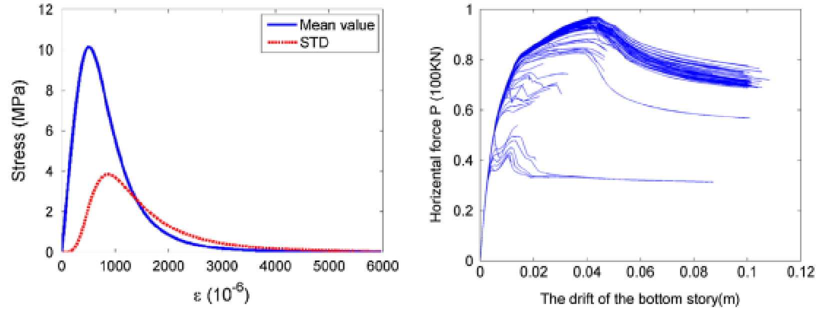




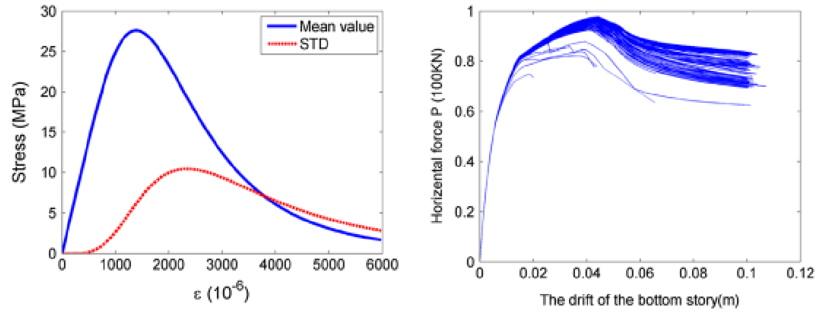
Column section Beam section



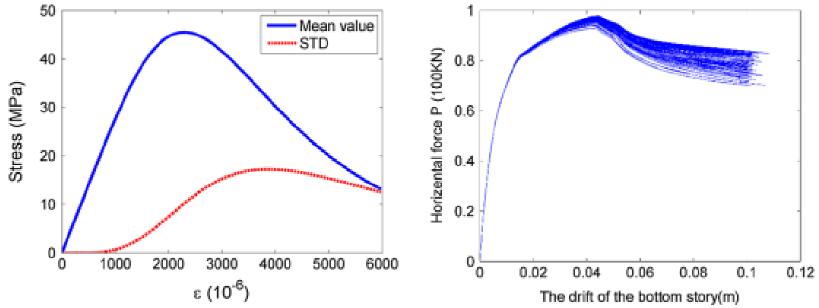




(a) $\mu_0 = 6.5$， $\sigma_0=0.5$， $L_0=0.1$



(b) $\mu_0 = 7.5$， $\sigma_0=0.5$， $L_0=0.1$

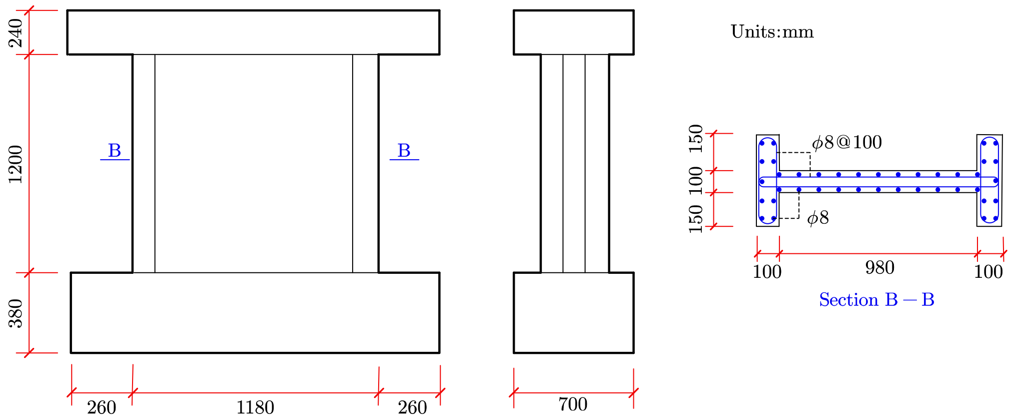


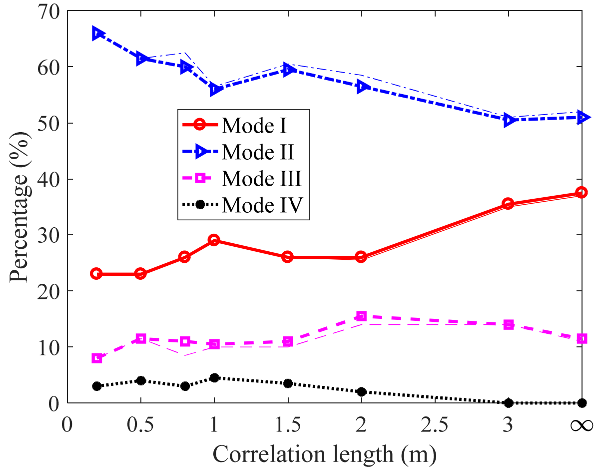
(c) $\mu_0 = 8.0$， $\sigma_0=0.5$， $L_0=0.1$

Ren XD, Yue QX. Structure and Infrastructure Engineering, 2018, 14(6), 780-790.

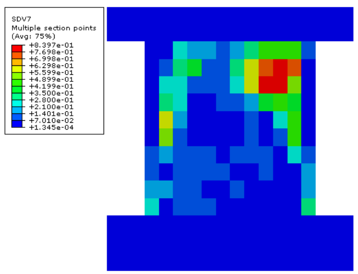
4.2 Stochastic Response Analysis

➢ Random transition of failure modes

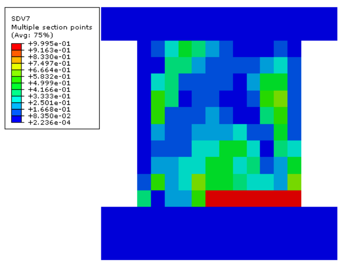




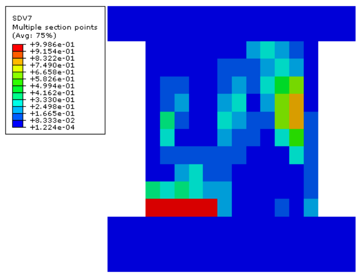
Compressive damage:



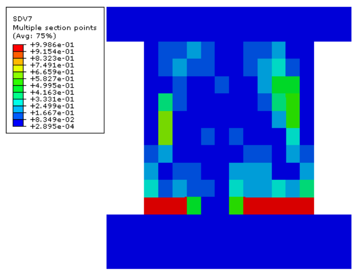
Mode I



Mode II



Mode III



Mode IV

4.2 Stochastic Response Analysis

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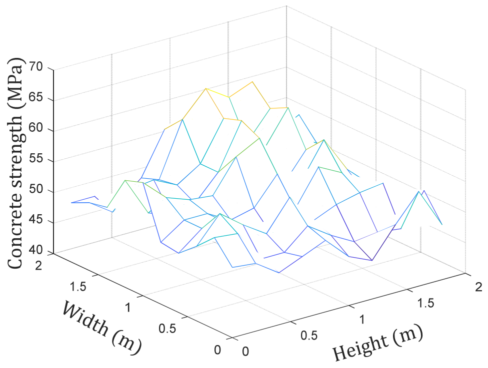
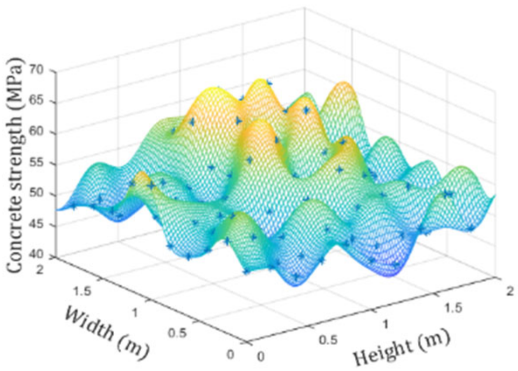
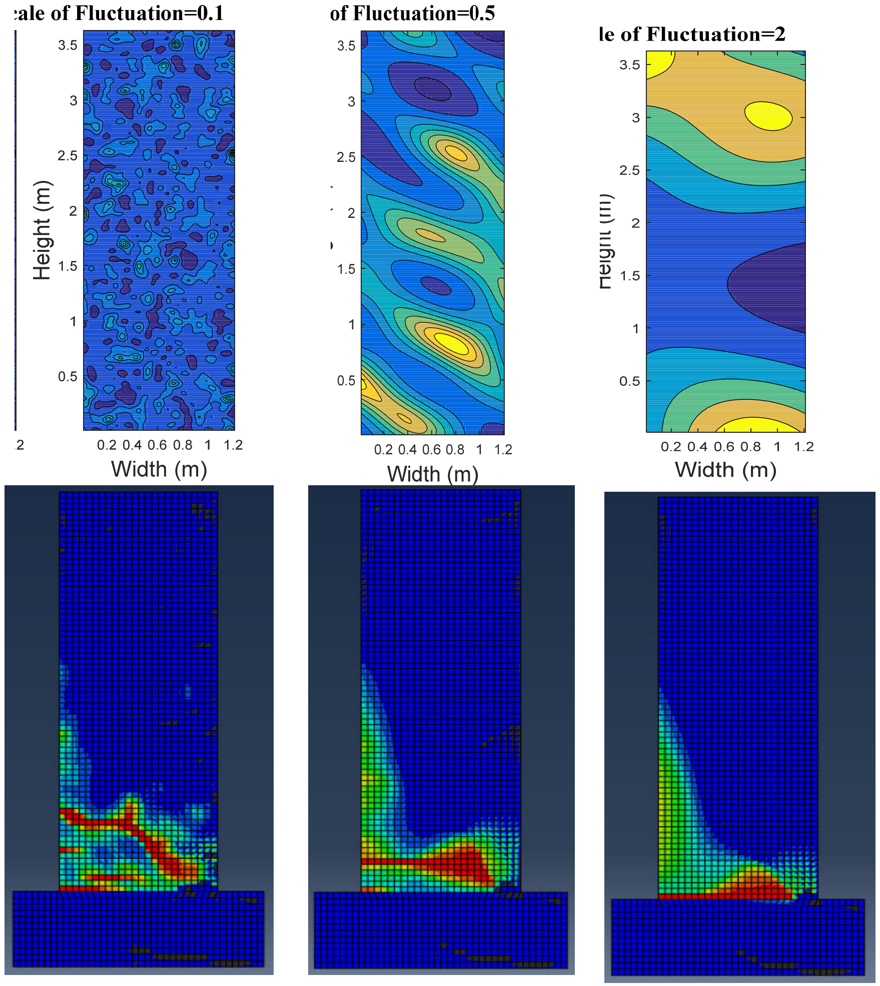
- Synthesizing the Bayesian compressive sensing (BCS) and the stochastic harmonic function (SHF)



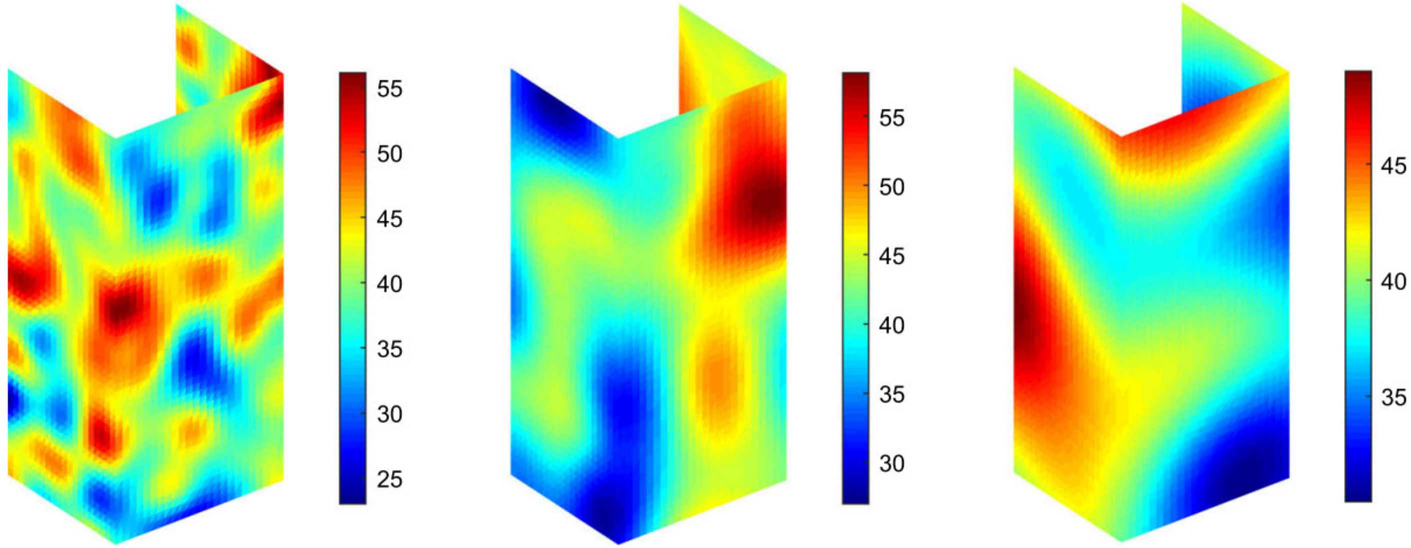


One sample

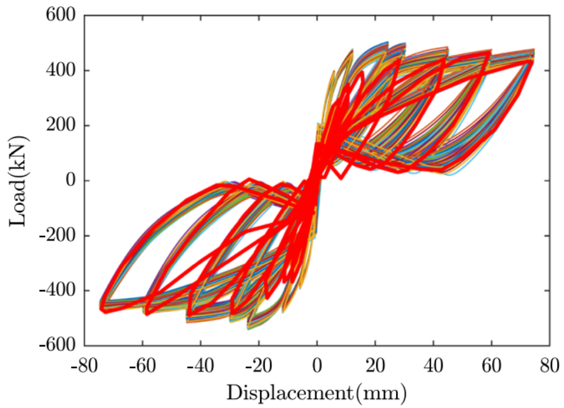


Chen JB, He JR, Ren XD, Li J. Mechanical Systems and Signal Processing, 2021, 153: 107377.

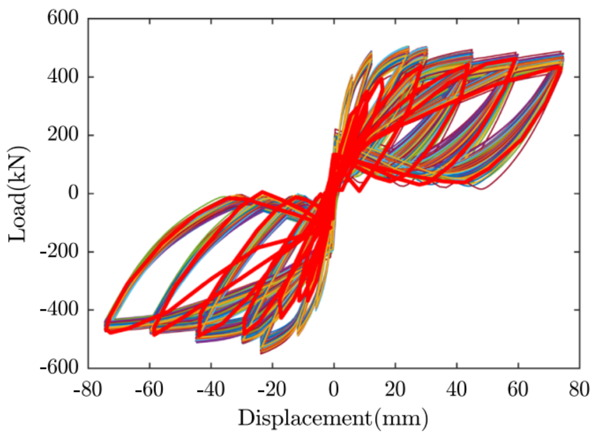
4.2 Stochastic Response Analysis



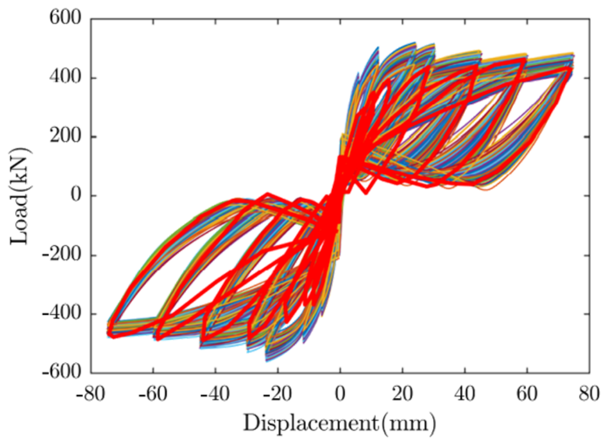
(a) c=0.5 m **(b)** c=1.0 m **(c)** c=2.0 m



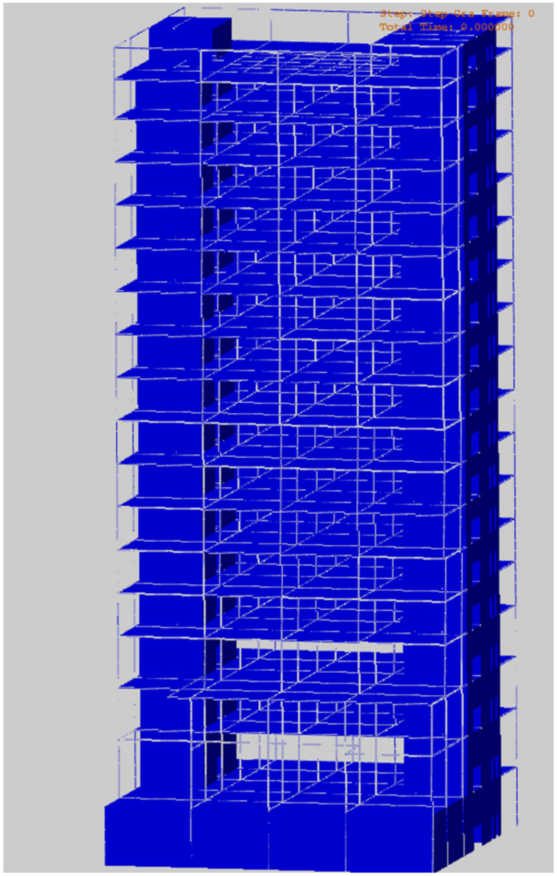
(a) c=0.5m

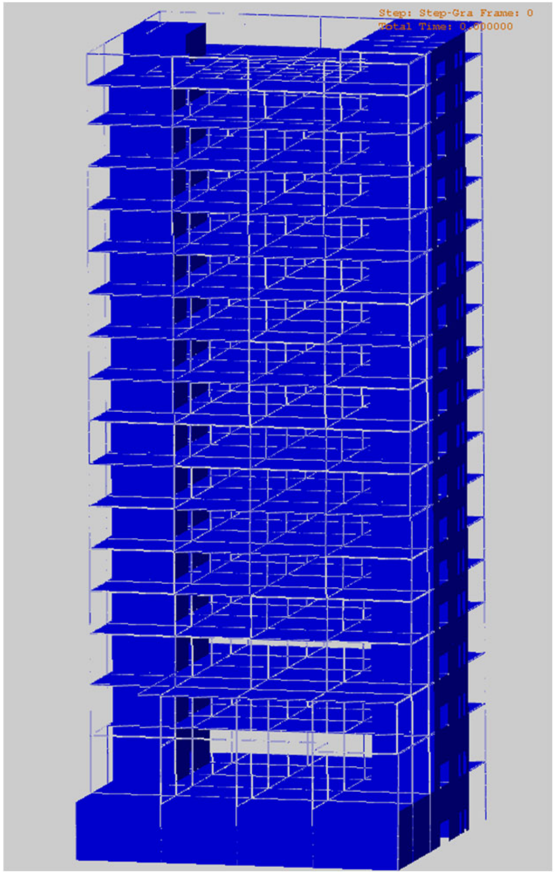


(b) c=1m



(c) c=2m





Randomness collapse of high-rise buildings

Zhou H, Li J, Ren XD. International Journal for Multiscale Computational Engineering, 2016, 14(3):303-321.

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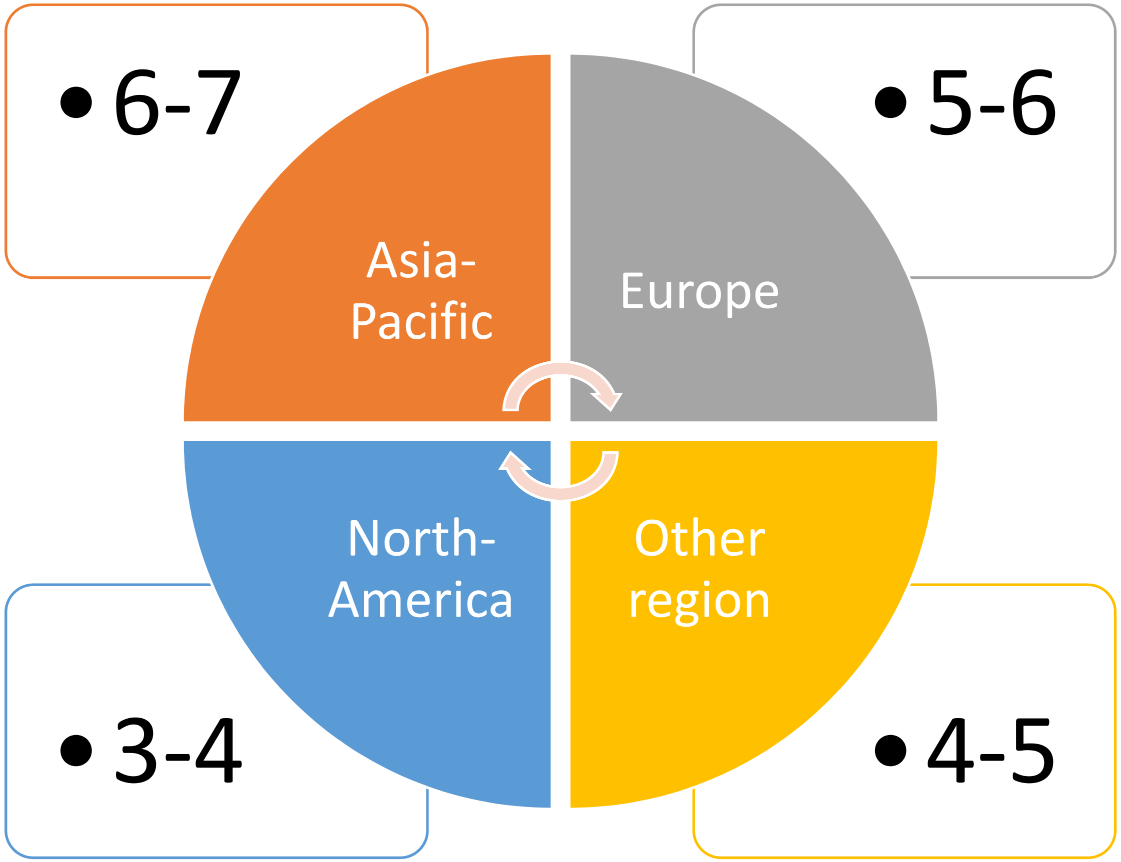
Contents

5 Plan of next move

➢ PMC concrete working group



20+members

5 Plan of next move

1. To accommodate the complete process of stress-strain curves with concept of damage
2. To consider the full-probabilistic joint distribution of model parameters
3. To consider both compressive and tensile stress-strain curves
4. To include particular models for multi-dimensional constitutive relationship (at least multi-axial strength)
5. To involve time-dependent effect or not?

The investigations have been done.

The report is in preparation.

The draft of main content will be start soon.

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Thanks for your attention!

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